1. **Possion Process:** (20%)
The p.d.f. of Possion distribution is

$$f(x) = \frac{\lambda^x e^{-\lambda}}{x!}. \tag{1}$$

Show how this distribution can be *derived* from the binomial distribution. Your answer should comprise of two parts. First, write down the **approximate Possion process** in terms of binomial distribution. Then, show that the limit of this binomial distribution is Possion distribution.
(Hint: The following formula may be useful for you.)

$$\lim_{n\to\infty}(1-\frac{\lambda}{n})^n = e^{-\lambda} \tag{2}$$

2. **Simulate a Random Variable:** (20%)
If your computer supplies you with only *uniform distribution*, show how to simulate observations sampled from an exponential distribution with a mean of $\theta = 10$. Note that the distribution function of X is

$$F(x) = 1 - e^{-\frac{x}{10}}, \quad 0 \le x < \infty. \tag{3}$$

3. **Normal Distribution** (20%)
Prove that

$$\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} exp[-\frac{(x-\mu)^2}{2\sigma^2}] = 1 \tag{4}$$

4. **Probabilistic Independence** (10%)
Let the joint probability density function of X and Y be

$$f(x,y) = \frac{xy^2}{30}, \tag{5}$$

where $x = 1, 2, 3$, and $y = 1, 2$. Show that X and Y are independent.

5. **Moment-Generating Function** (10%)
Let X_1 and X_2 have independent distribution $b(n_1, p)$ and $b(n_2, p)$. Find the moment-generating function of

$$Y = X_1 + X_2. \tag{6}$$

How is Y distributed?

6. **Maximum Likelihood Estimator** (10%)
Let

$$f(x;\theta) = \theta x^{\theta-1}, \quad 0 < x < 1, \tag{7}$$

where $\theta \in \Omega = \{\theta : 0 < \theta < \infty\}$. Let $X_1, X_2, ..., X_n$ denote a random sample of size n from this distribution. Find the *maximum likelihood estimator* of θ.

7. **Confidence Interval** (10%)
Let $X_1, X_2, X_3, ..., X_n$ be a random sample from $N(\mu, \sigma^2)$, with known mean μ. Describe how you would construct a confidence interval for the unknown variance σ^2.