

## 國立臺北科技大學 108 學年度碩士班招生考試

系所組別：1111、1112 機械工程系機電整合碩士班甲組

## 第一節 工程數學 試題

第一頁 共一頁

**注意事項：**

1. 本試題共 5 題，每題 20 分，共 100 分。
2. 不必抄題，作答時請將試題題號及答案依照順序寫在答案卷上。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

5. Please solve the following PDE. (20 %)

$$\text{PDE} \quad \frac{\partial u(x,t)}{\partial t} = \frac{\partial^2 u(x,t)}{\partial x^2} \quad (-1 < x < 1, \quad t \geq 0)$$

$$\text{BC} \quad \begin{cases} u(-1,t) = 2 \\ u(1,t) = 4 \end{cases} \quad (t \geq 0)$$

$$\text{IC} \quad u(x,0) = 3 + x + \sin(2\pi x) \quad (-1 < x < 1)$$

1. Please solve the following differential equation: (20%)

(1)  $x^2 y'' + 5xy' + 13y = 0$  (10%)

(2)  $y'' + 2xy' + x^2 y = 0$  (10%)

2. (1) If **A** and **B** are **invertible matrix**, please prove that  $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$ . (10 %)

(2) Find the eigenvalue and corresponding eigenvector of  $A = \begin{pmatrix} -5 & 2 \\ 2 & -2 \end{pmatrix}$ . (10 %)

3. Let  $V$  describe the region bounded by the hemisphere  $x^2 + y^2 + (z-2)^2 = 9$ ,  $2 \leq z \leq 5$ ,and the plane  $z = 2$ . Please verify the divergence theorem if  $\vec{F} = x\vec{i} + y\vec{j} + (z-2)\vec{k}$ .

(20%)

4. (1) Please prove that  $\mathcal{L}\{e^{at}f(t)\} = F(s-\alpha)$  ( $s-\alpha > k > 0$ ),  $\mathcal{L}\{\cdot\}$  denotes the Laplacetransform from  $t$  to  $s$ . (10 %)

(2) Please find the inverse Laplace transform of  $\frac{se^{-10s}}{(s^2+9)^2}$ . (10 %)