

國立高雄科技大學 108 學年度碩士班 招生考試 試題紙

系所別：電腦與通訊工程系碩士班

組別：甲組

考科代碼：2031

考科：工程數學

注意事項：

- 1、各考科一律可使用本校提供之電子計算器，考生不得使用自備計算器，違者該科不予計分。
- 2、請於答案卷上規定之範圍作答，違者該題不予計分。

1. [10%] Solve  $\frac{dy}{dx} = \frac{2x(y+1)}{x^2+4}$ .

2. [10%] Solve  $x^2 \frac{dy}{dx} + 2xy = 3x^2$ .

3. [10%] Solve  $y'' + 2y' + 3y = 0$ .

4. [10%] Find the Laplace transform of  $f(t) = \begin{cases} 2, & 0 \leq t < 3 \\ 0 & t \geq 3 \end{cases}$ .

5. [10%] Solve  $f(t) = 3e^{2t} + \int_0^t f(t-\tau)e^{2\tau} d\tau$  for  $f(t)$ .

6. [10%] Let  $A = \begin{bmatrix} 2 & -1 \\ -3 & 4 \end{bmatrix}$  and  $B = \begin{bmatrix} 1 & 5 \\ 2 & 3 \end{bmatrix}$

(a) Compute  $AB$ . (5%)

(b) Compute  $A^T + 2B$ . (5%)

7. [10%] Let  $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 3 & 1 \\ 0 & 3 & 0 \end{bmatrix}$  and  $B = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 0 & 1 \\ 0 & 3 & 0 \end{bmatrix}$ .

(a) Find an elementary matrix  $E$  such that  $EA = B$ . (5%)

(b) Find the inverse of  $B$ . (5%)

8. [10%] Let  $A = \begin{bmatrix} 2 & 1 & -2 & 1 \\ 1 & 3 & 4 & 8 \\ 1 & 1 & -1 & 1 \end{bmatrix}$ .

- (a) Find the rank of  $A$ . (5%)
- (b) Find a basis for the nullspace of  $A$ . (5%)

9. [20%] Let  $A = \begin{bmatrix} 0 & -2 & 1 \\ -2 & 3 & -2 \\ 1 & -2 & 0 \end{bmatrix}$ . (Hint:  $\lambda = -1$  is an eigenvalue of  $A$ )

- (a) Find the eigenspace for  $A$  corresponding to the eigenvalue  $\lambda = -1$ . (5%)
- (b) Find all eigenvalues for  $A$  (other than  $\lambda = -1$ ) 除了  $\lambda = -1$  以外的其他特徵值。 (5%)
- (c) Find a nonsingular matrix  $X$  and a diagonal matrix  $D$  such that  $A$  can be factored into a product  $A = XDX^{-1}$ . (10%)