1. (10 %) Two sections of a senior probability course are being taught. From what she has heard about the two instructors listed. Lisa estimates that her chances of passing the course are 0.85 if she gets professor X and 0.60 if she gets professor Y. The section into which she is put is determined by the registrar. Suppose that her chances of being assigned to professor X are 4 out of 10. Fifteen weeks later we learn that Lisa did, indeed, pass the course. What is the probability she was enrolled in professor X's section?

2. (10%) In a sample of *25*, $\bar{x} = 1.63$ and $s = 0.51$.
Construct a *95* percent confidence interval for μ. Assuming that the sample of 25 is from a normal distribution, $\text{Prob}\left(-2.064 \le \frac{5(\bar{x}-\mu)}{s} \le 2.064\right) = 0.95$,
where 2.064 is the critical value from a t distribution with 24 degrees of freedom.

3. (15%) Let *X* be a random variable of the continuous type with the probability density function *f(x)*, which is positive provided $0 < x < b < \infty$, and is equal to zero elsewhere. Show that $E(X) = \int_0^b [1 - F(x)]dx$, where *F(x)* is the distribution function of *X*.

4. (15%) Prove that *if* $x_1, \cdots, x_n$ *are a random sample from a population with mean* μ *and variance* σ^2, *then* $\bar{x}$ *is a random variable with mean* μ *and variance* σ^2 / n.

5. (15%) Let Y_1, Y_2 and Y_3 be a random sample from a normal distribution where both μ and σ^2 are unknown. Which is a more efficient estimator for μ,

$$\hat{\mu}_1 = \frac{1}{4}Y_1 + \frac{1}{2}Y_2 + \frac{1}{4}Y_3$$

or

$$\hat{\mu}_2 = \frac{1}{3}Y_1 + \frac{1}{2}Y_2 + \frac{1}{6}Y_3$$

6. (15%) Among the problems faced by job seekers wanting to reenter the work force, eroded skills and outdated backgrounds are two of the most difficult to overcome. Knowing that, employers are often wary of hiring individuals who have spent lengthy periods of time away from the job. The following table shows the percentages of hospitals willing to rehire medical technicians who have been away from that career for x years. It can be argued that the fitted line should necessarily have a y-intercept of 100 because no employer would refuse to hire someone (due to outdated skills) whose career had not been interrupted at all--that is, applicants for whom $x = 0$. Under that assumption, estimate β_1 by fitting these data with the model $y = 100 + \beta_1 x$.

Years of Inactivity, x	Percent of Hospitals Willing to Hire, y
0.5	100
1.5	90
4	75
8	44
13	28
18	17

7. (20%) A useful measure of a stock's profitability is its yield, defined to be its dividends for the previous 12 months times 100, divided by its current market value. The following table gives the yields of the New York Stock Exchange Common Stock Index for each quarter of the years 1981 through 1985. Are yields affected by the quarter of the year? Is the variability in the yield from year to year statistically significant? Construct the ANOVA table and state your conclusions using the α = 0.05 level of significance.

	Quarter			
Year	First	Second	Third	Fourth
1981	5.7	6.0	7.1	6.7
1982	7.2	7.0	6.1	5.2
1983	4.9	4.1	4.2	4.4
1984	4.5	4.9	4.5	4.5
1985	4.4	4.2	4.2	3.6

($F_{0.05,3,12} = 3.4903$, $F_{0.05,4,12} = 3.25916$, $F_{0.05,5,12} = 3.105875$, $F_{0.05,3,20} = 3.098393$, $F_{0.05,4,20} = 2.866081$, $F_{0.05,5,20} = 2.710891$)