考試科目	微積分	所別	財政學系	考試時間	月 日 星期 日 下午 第4節

1. Find the values of *a* and *b*, if the limit $\lim_{x\to 0}\frac{\sqrt{1+x+x^2}-(1+ax)}{x^2}=b$ exists. **(5 points)**

2. Let $F(x,y)=y-x^y=0$. Find $\frac{dy}{dx}$. **(5 points)**

3. Sketch the graph of $y=\frac{x}{x^2+1}$. Label the intercepts, relative extrema, points of inflection, and asymptotes. Then state the domain of the function. **(10 points)**

4. Evaluate the definite integral $\int_{-1}^{e}\ln|x|\,dx$. **(5 points)**

5. After test-marketing a new menu item, a fast-food restaurant predicts that sales of a new item will grow according to the model $\frac{dS}{dt}=\frac{2t}{(t+4)^2}$, where *t* is the time in weeks and *S* is the sales in thousands of dollars. Find the sales of the menu item at 10 weeks.
[Hints: ln 2 = 0.6931, ln 3 = 1.0986, ln 5 = 1.6094, ln 7 =1.9459, ln 11 = 2.3978.] **(10 points)**

6. Examine the function $f(x,y)=-\frac{4x}{x^2+y^2+1}$ for the relative exterma and saddle points.
(10 points)

7. An investor is considering three different stocks in which to invest $20,000. The average annual dividends for the stocks are

General Motors (G)	3.0%
Eastman Kodak (E)	2.3%
Kelly Services, Inc. (K)	2.5%

The amount invested in Eastman Kodak must follow the equation

$$2000K-2000G+E^2=0$$

How much should be invested in each stock to yield a maximum of dividends? **(10 points)**

8. Find the value of *a* and *b* such the linear model $f(x)=a+bx$ has a minimum sum of the squared errors for the points (-3, 0), (-1, 1), (0, 2), (2, 3). **(10 points)**

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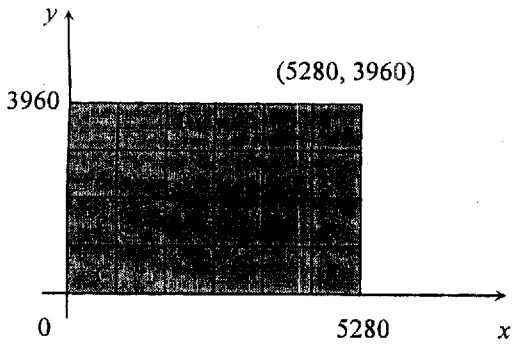
9. The average value of real estate (in dollars per square foot) for a rectangular section of a city is given by $f(x, y) = 2.5x^{3/2}y^{3/4}$ (see figure). Find the average value of real estate for this section. (5 points)



10. The ordering and transportation cost C of the components used in manufacturing a product is

$$c = 100\left(\frac{200}{x^2} + \frac{x}{x+30}\right), x \geq 1$$

where C is measured in thousands of dollars and x is the order size in hundreds. Find the order size that minimizes the cost. [Hint: Use Newton's Method.] (10 points)

11. Use a sixth-degree Taylor polynomial centered at zero to approximate the definite integral $\int_0^{1/2} e^{-x^2}\,dx$. (5 points)

12. A large corporation starts at time $t = 0$ to invest part of its receipts at a rate of P dollars per year in a fund for future corporate expansion. Assume that the fund earns r percent interest per year compounded continuously. So, the rate of growth of the amount A in the fund is given by $\frac{dA}{dt} = rA + P$, where $A = 0$ when $t = 0$. Solve this differential equation for A as a function of t. (5 points)

13. Let $A(t)$, known as the area function, denote the area of the region R under the graph of a nonnegative function $y = f(x)$ from $x = a$ to $x = t$, where $a \leq t \leq b$ (see figure). Use the concept of area function to demonstrate the plausibility of the fundamental theorem of calculus for the case where f is nonnegative on an interval $[a, b]$. (10 points)