● 可使用現行『國家考試電子計算器規格標準』規定第一類之計算機

※※Please provide the detailed process or you will get zero point※※

1. (24%) Differentiate the given functions. (8% for each item)

   (a) $g(u) = \ln(u + \sqrt{u^2 + 1})$

   (b) $f(x) = (1 + x)^{\sqrt{1+x}}$

   (c) $f(x) = \dfrac{x^3(2x^2-5)^2\sqrt{10x^4-3x}}{\sqrt[3]{4x^3-25}}$

2. (40%) Find the indicated integrals. (8% for each item)

   (a) $\int \dfrac{e^x + e^{-x}}{e^x - e^{-x}} dx.$

   (b) $\int \dfrac{2x\ln(x^2+1)}{x^2+1} dx.$

   (c) $\int_{-\infty}^{+\infty} xe^{-x} dx.$

   (d) $\int_{1}^{+\infty} \dfrac{e^{-\sqrt{x}}}{\sqrt{x}} dx.$

   (e) $\int_{1}^{e} \int_{0}^{\ln x} xy\, dy\, dx$

3. (14%) Please approximate the given integral, $\int_{0}^{1} \sqrt[3]{1 + x^2}\, dx, n = 4$, using (a) the trapezoidal rule and (b) Simpson's rule with the specified number of subintervals. (7% for each item)

4. (10%) In the following question, $f(x, y) = xy^2 - 6x^2 - 3y^2$, find the critical points of the given functions and classify each as a relative maximum, a relative minimum, or a saddle point.

5. (12%) When x thousand dollars are spent on labor and y thousand on equipment, the output of a certain factory is Q units, where $Q(x, y) = 60x^{1/3}y^{2/3}$. Suppose $120,000 is available for labor and equipment. (a) (10%) How should the money be allocated between labor and equipment to generate the largest possible output? (b) (2%) Use the Lagrange multiplier λ to estimate the change in the maximum output of the factory that will result if the money available for labor and equipment is increased to $121,000.

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