(一)

一. Let $V_1 = \left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} \middle| \; b > 0 \right\}$, $V_2 = \left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} \middle| \; a - b + c = 0 \right\}$.

Are V_1 and V_2 subspaces of R^3? Explain. [10%]

二. Let A be a 4x5 matrix.

(a) If the rank of A is 3, what is the dimension of its column space? Explain. [6%]

(b) If the rank of A is 3, what is the dimension of the solution space of the homogeneous system $Ax = 0$? Explain. [6%]

三. Let $L: R^4 \to R^3$ be defined by

$$L\left(\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix}\right) = \begin{bmatrix} a_1 + a_2 \\ a_3 + a_4 \\ a_1 + a_3 \end{bmatrix}.$$

(a) Is L a linear transformation? Justify your answer. [6%]

(b) Find a basis for $\ker L$. [6%]

(c) Find a basis for range L. [6%]

(二)

1. (12%) When we encode members of Z_2^m into Z_2^n; such a code is called an (m, n) code. The first m components of a code word are the information digits, and the last r (r = n – m) components are the check digits. Please answer (1) which of the following are perfect codes, and (2) which are single-error correcting codes?

   (a) (12, 7) (b) (15, 11) (c) (5, 3)

2. (12%) How many nonnegative integer solutions are there to the inequality $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 < 10$.

3. (12%) Please solve the recurrence relation $\begin{cases} a_{n+2} = 4\,a_{n+1} - 4a_n,\ n \geq 0 \\ a_0 = 1,\ a_1 = 3 \end{cases}$

4. (12%) Let f : S → T, and g : T → U. Then the composition function, g ∘ f, is a function from S to U defined by (g ∘ f)(s) = g(f(s)). Consider the following statements, which are false statement(s)?

   (a) If f is one-to-one then f ∘ g is one-to-one.
   (b) If f and g are onto then f ∘ g is onto.
   (c) If f and g are one-to-one and onto then f ∘ g is one-to-one and onto.
   (d) If f ∘ g is one-to-one then f is one-to-one.
   (e) If f ∘ g is one-to-one then g is one-to-one.
   (f) If f ∘ g is onto then f is onto.
   (g) If f ∘ g is onto then g is onto.

5. (12%) Please (1) design a minimum rail network connecting the seven cities, shown in the mileage chart below, (2) show sum of the network.

	City-1	City-2	City-3	City-4	City-5	City-6	City-7
City-1	0	500	400	600	100	550	300
City-2	500	0	620	1100	450	1000	700
City-3	400	620	0	525	520	900	420
City-4	600	1100	525	0	700	430	200
City-5	100	450	520	700	0	490	350
City-6	550	1000	900	430	490	0	330
City-7	300	700	420	200	350	330	0