(一)

1. (12%) When we encode members of $Z_2^m$ into $Z_2^n$; such a code is called an (m, n) code. The first m components of a code word are the information digits, and the last r (r = n – m) components are the check digits. Please answer (1) which of the following are perfect codes, and (2) which are single-error correcting codes?

   (a) (12, 7) (b) (15, 11) (c) (5, 3)

2. (12%) How many nonnegative integer solutions are there to the inequality $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 < 10$.

3. (12%) Please solve the recurrence relation $\begin{cases} a_{n+2} = 4a_{n+1} - 4a_n, n \geq 0 \\ a_0 = 1, a_1 = 3 \end{cases}$

4. (12%) Let f : S → T, and g : T → U. Then the composition function, g ∘ f, is a function from S to U defined by (g ∘ f)(s) = g(f(s)). Consider the following statements, which are false statement(s)?

   (a) If f is one-to-one then f ∘ g is one-to-one.
   (b) If f and g are onto then f ∘ g is onto.
   (c) If f and g are one-to-one and onto then f ∘ g is one-to-one and onto.
   (d) If f ∘ g is one-to-one then f is one-to-one.
   (e) If f ∘ g is one-to-one then g is one-to-one.
   (f) If f ∘ g is onto then f is onto.
   (g) If f ∘ g is onto then g is onto.

5. (12%) Please (1) design a minimum rail network connecting the seven cities, shown in the mileage chart below, (2) show sum of the network.

| | City-1 | City-2 | City-3 | City-4 | City-5 | City-6 | City-7 |
|---|---|---|---|---|---|---|---|
| City-1 | 0 | 500 | 400 | 600 | 100 | 550 | 300 |
| City-2 | 500 | 0 | 620 | 1100 | 450 | 1000 | 700 |
| City-3 | 400 | 620 | 0 | 525 | 520 | 900 | 420 |
| City-4 | 600 | 1100 | 525 | 0 | 700 | 430 | 200 |
| City-5 | 100 | 450 | 520 | 700 | 0 | 490 | 350 |
| City-6 | 550 | 1000 | 900 | 430 | 490 | 0 | 330 |
| City-7 | 300 | 700 | 420 | 200 | 350 | 330 | 0 |

| 科目 | 計算機數學(一) | 所別 | 資訊科學研究所<br>碩士班 | 考試時間 | 4月18日上午第3節<br>星期日 下 |
|---|---|---|---|---|---|



(二)

（機率部分的題目每一題都需要寫出計算過程）

1 (12%) 令 f(x)=cx(1-x)是 random variable X 的 probability density function，其中 c 為一個常數。若 X 的分佈區間是[0,1]，回答下列問題。

1.1 (4%) 求 c 的值

1.2 (4%) 令 Y=3X+2，求 E[Y] 的值

1.3 (4%) 求 VAR[X]的值

2 (13%) 已知變數 X 和 Y 之間的關係是 $X=aY^b$,假定我們已經獲知在 $Y=y_i$ 時，$X=x_i$，i=1,2,3,4 四組數據，利用這四組抽樣數據求出和表示 a 和 b 的值。

3 (15%) 假定莊家有甲和乙兩個骰子，甲乙兩骰子擲出數字的機率如下表。在任一賭局中莊家只會使用甲乙兩骰子的其中一個，並且會先試擲其所使用的骰子三次。在某一次賭局中，莊家在試擲時擲出 1，4 和 6。若我們假設每一次試擲均為獨立事件，並且假設莊家使用甲乙兩個骰子的機率各為 0.6 和 0.4，在該次賭局中，莊家使用甲乙兩骰子的機率各為多少？

| 數字 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 甲骰子 | 0.1 | 0.2 | 0.2 | 0.1 | 0.3 | 0.1 |
| 乙骰子 | 0.2 | 0.1 | 0.1 | 0.3 | 0.1 | 0.2 |