| 考試科目 | 微積分 | 所別 | 應用數學系 | 考試時期 | 4月17日 星期六 | 下午第一節 |
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1. (a) Evaluate the integral $\iint_{R_a} e^{-(x^2+y^2)}dxdy$, where $a$ is a positive constant and

   $R_a = \{(x,y) \in \Re^2 \mid x^2+y^2 \le a^2\}$. (8%)

   (b) Use (a) to evaluate the improper integral $\int_{-\infty}^{\infty} e^{-x^2}dx$. (8%)

2. Let $f:\Re \to \Re$ be a differentiable function such that $0 \le f'(x) \le f(x)$, $\forall x \in \Re$. Show that

   (a) the function $g(x) = e^{-x} f(x)$ is nonincreasing. (8%)

   (b) if $f$ vanishes at some point, then $f \equiv 0$ on $\Re$. (8%)

3. Let $\{a_n\}$ be a real sequence. Prove or disprove the following statements:

   (a) If $\sum |a_n|$ converges, then $\sum a_n^2$ converges. (8%)

   (b) If $\sum a_n^2$ converges, then $\sum |a_n|$ converges. (8%)

4. (a) State the mean-value theorem for derivatives. (8%)
   (b) Use (a) to deduce the inequality $|\cos x - \cos y| \le |x-y|$, $\forall x, y \in \Re$. (8%)

5. For each $n = 1, 2, \cdots$, let $f_n(x) = x^n$, $0 \le x \le 1$.

   (a) Prove that the sequence $\{f_n\}$ converges pointwise on $[0,1]$. (8%)
   (b) Does $\{f_n\}$ converge uniformly on $[0,1]$? Justify your answer. (8%)

6. Use Lagrange's multiplier to prove that the minimum distance from a point $(x_0, y_0, z_0)$ to a plane $ax+by+cz+d=0$ is $\dfrac{|ax_0+by_0+cz_0+d|}{\sqrt{a^2+b^2+c^2}}$. (10%)

7. Evaluate the line integral $\oint_C xy^3dx + 2x^2y^2dy$, where $C$ denotes the boundary of the region in the first quadrant enclosed by the $x$-axis, the line $x=1$, and the curve $y=x^3$. (10%)

| 備考 | 試題隨卷繳交 |
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命題老師： -54- （簽章） 92 年 3 月 21 日