考試科目	線性代數	所別	應用數學系	考試時期	4月17日 星期六	下午第二節

1. Let $P_2 = \{ax^2 + bx + c \mid a,b,c \in \Re\}$ and $T : P_2 \to P_2$ be a mapping defined by $T(ax^2 + bx + c) = 2cx^2 - ax + b$ for all $ax^2 + bx + c \in P_2$.
   (a) Show that T is a linear transformation. (5%)
   (b) Is T an isomorphism of P_2? (5%)
   (c) Find the matrix representation $[T]_\alpha$ of T with respect to the ordered basis $\alpha = \{x^2, x, 1\}$. (5%)
   (d) Find the determinant of T. (5%)

2. Let S be the vector subspace in $\Re^4$ spanned by $\{(1,0,1,0),(0,1,0,1)\}$ and $v = (1,2,3,4) \in \Re^4$.
   (a) Is $v \in S$? (5%)
   (b) Find the orthogonal projection of v onto S. (10%)
   (c) Find the minimum distance from v to S. (10%)

3. (a) Let A be a nonsingular (invertible) $n \times n$ real matrix. Show that the inverse matrix A^{-1} of A can be expressed as a polynomial in A, that is,

$$A^{-1} = a_k A^k + a_{k-1} A^{k-1} + \cdots + a_1 A + a_0 I,,$$

   for some positive integer k and $a_0, \cdots, a_k \in \Re$. (10%)

   (b) Let $A = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}$.

   Express A^{-1} as a polynomial in A. (5%)

4. Let A be an $n \times n$ square matrix of rank one. Show that $A^2 = \alpha A$ for some $\alpha \in \Re$. (10%)

5. Let A be an $n \times n$ real symmetric matrix. Show that
   (a) All eigenvalues of A are real. (5%)
   (b) Eigenvectors of A corresponding to distinct eigenvalues are orthogonal. (5%)
   (c) A is positive definite if and only if $A = B^T B$ for some nonsingular matrix B. (10%)

6. Identify the conic $x^2 + 4xy + y^2 + 3x + y - 1 = 0$ and transform the conic into standard form. (10%)

備考	試題隨卷繳交

命題老師：　　(簽章) 93年3月30日