

國立臺灣師範大學 108 學年度碩士班招生考試試題

科目：工程數學

適用系所：機電工程學系

注意：1.本試題共 2 頁，請依序在答案卷上作答，並標明題號，不必抄題。2.答案必須寫在指定作答區內，否則依規定扣分。

1. (10 分) Consider the Sturm-Liouville problem

$$y'' + \lambda y = 0; \quad y(0) = y(1) = 0$$

on an interval $[0, 1]$. Find numbers λ and corresponding nontrivial solutions of the Sturm-Liouville equation on $[0, 1]$.

2. (10 分) Define a function $f(x) = \sin x - 1$ on the interval $(-\pi, \pi)$. Determine the Fourier series of $f(x)$ on the interval $(-\pi, \pi)$.

3. (10 分) Let C be a smooth curve with coordinate functions $x = 4 \cos t$, $y = 4 \sin t$, $z = -3$ for $0 \leq t \leq \pi/2$. Evaluate $\int_C xy \, ds$ over C , in which s denotes arc length.

4. (10 分) Let $\varphi(x, y, z) = 8xy^2 - xz$. Compute the directional derivative of the scalar field φ in the direction of $\mathbf{i} + \mathbf{j} + \mathbf{k}$.

5. (10 分) Consider the system $x'(t) = 3x(t) - 5y(t)$, $y'(t) = 5x(t) - 7y(t)$. Classify the critical point of the system as to being “stable and asymptotically stable,” “stable and not asymptotically stable,” or “unstable.”

6. (15 分) Let $\mathbf{A} = \begin{bmatrix} 5 & 0 & 2 \\ 0 & 0 & 0 \\ -2 & 0 & 0 \end{bmatrix}$. Find a matrix \mathbf{P} such that $\mathbf{P}^{-1}\mathbf{A}\mathbf{P}$ is a diagonal matrix.

國立臺灣師範大學 108 學年度碩士班招生考試試題

7. (15 分) Two objects of masses m_1 and m_2 are attached to opposite ends of a spring having spring constant k , as shown in Figure 1. The entire apparatus is placed on a highly varnished table. Derive differential equations to model the system sliding in one dimension in the absence of friction. Then, find the frequency of oscillation of the masses with respect to each other when they are stretched and released from rest.

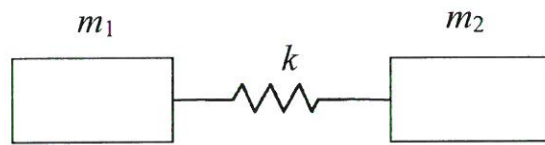


Figure 1. System for Problem 7

8. (10 分) Let $\delta(t)$ denote the delta function. Solve the second-order differential equation $y''(t) + 2y'(t) + 2y(t) = \delta(t - 3)$ subject to $y'(0) = y(0) = 0$.
9. (10 分) In the circuit of Figure 2, the current flowing through the inductor is initially zero. How long after the switch is closed will the inductor voltage be 6 volts? Determine the current in the resistor at that time.

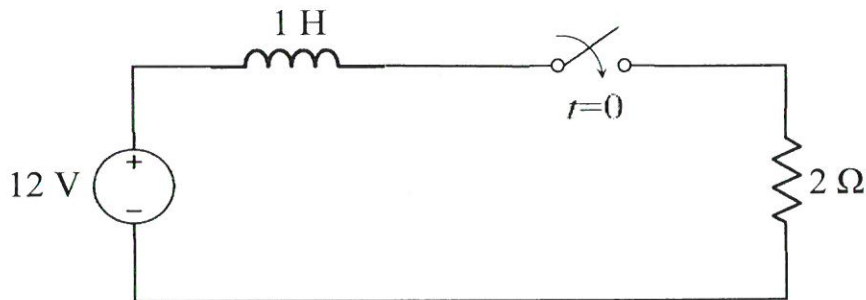


Figure 2. Network for Problem 9