

# 淡江大學 108 學年度碩士班招生考試試題

系別：航空太空工程學系

科目：工程數學

13-1

考試日期：3 月 10 日(星期日) 第 1 節

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1. Consider the following ordinary differential equation of initial value problem,

$$y'' + 5y' + 6y = 3x \quad (1)$$

where  $y$  is differentiated with respect to  $x$ . Let  $y(0) = 1$  and  $y'(0) = 1$ , please answer the following questions:

- (a) (20 %) Please solve this problem by assuming  $y_h(x) = e^{\lambda x}$  along with the method of undetermined coefficients, where  $y_h(x)$  denotes a homogeneous solution.  
(b) (20 %) Please solve this problem by Laplace Transform.

2. Consider the following ordinary differential equation of initial value problem,

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} \quad (2)$$

where  $\mathbf{x} = [x_1 \ x_2]^T$  and

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -8 & -6 \end{bmatrix}$$

with initial conditions  $x_1(0) = -1$  and  $x_2(0) = 1$ .

- (a) (20 %) Please find the eigenvalues and the corresponding eigenvectors of matrix  $\mathbf{A}$ . Remember to represent the eigenvectors as unit vectors.  
(b) (20 %) Please solve this ODE problem.

3. (20 %) A one-dimensional heat problem is described by a partial differential equation (PDE), given by

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2} \quad (3)$$

where  $u(x, t)$  denotes the temperature in the location  $x$  at time  $t$ , and  $c$  is a constant. Given the boundary conditions:  $u(0, t) = 0$ ,  $u(L, t) = 0$  for all  $t$ , and the initial condition  $u(x, 0) = \sin(x\pi/L)$ , please solve the PDE for  $u(x, t)$ .