

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. [10%] 多選題: Which of the following IVP has a unique solution?

(a) $(y - 1) \frac{dy}{dx} = 6x, y(0) = 1$

(b) $(y^2 + 1) \frac{dy}{dx} = x^2, y(3) = 1$

(c) $\sqrt{y - 1} y' = x, y(0) = 1$

(d) $x^2 y'' - 4xy' + 4y = 0, y(3) = 1, y'(3) = -1$

(e) $3y'' - 4y' + 1 = e^x, y(1) = 0, y'(1) = 1$

2. [10%] 是非題:

(1) In any vector space, $a\vec{x} = a\vec{y}$ implies $\vec{x} = \vec{y}$.

(2) If V is a vector space other than the zero vector space, V contains a subspace W such that $W \neq V$.

(3) Each element of a set S is a linear combination of the other elements of S if S is a linearly dependent set.

(4) A subset of a linearly independent set is also linearly independent

(5) The span of an empty set \emptyset is also \emptyset .

3. [15%] Solve the IVP: $\left(\frac{e^{-2\sqrt{x}-y}}{\sqrt{x}}\right) \frac{dx}{dy} = 1, y(1) = 1$

4. [10%] Solve the ODE: $(3x + 2)^2 y'' + 4(3x + 2)y' - 6y = \frac{1}{3x+2}$

5. [15%] Solve the IVP: $y'' + 9y = \mathcal{U}\left(t - \frac{\pi}{3}\right) \cos(3t); y(0) = 0; y'(0) = 1; \mathcal{U}(t)$ is a unit step function.

6. [10%] Solve the equation: $X^2 - 3X + 4I = \begin{bmatrix} 5 & -3 \\ -3 & 5 \end{bmatrix}$

7. Given a matrix $A = \begin{bmatrix} -6 & 19 & -2 & -16 \\ -40 & 0 & -40 & 20 \\ -8 & -8 & 14 & 12 \end{bmatrix}$

(1) [5%] Find the Null Space $N(A)$ of the matrix

(2) [5%] Find the singular values of the matrix A

(3) [10%] Find the Moore-Penrose Pseudoinverse of A

8. [10%] Prove that the eigenvectors corresponding to the distinct eigenvalues of a symmetric matrix A are orthogonal.