

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Consider the equation $y'' + 5y' + 6y = u(t-1) - u(t-2)$, $y(0) = 1$, and $y'(0) = 0$,

where $u(t-a) = \begin{cases} 0 & 0 \leq t < a, \\ 1 & a \leq t. \end{cases}$ Solve for $y(t)$ with Laplace transform. (20 points)

2. (a) Find the tangent to the ellipse $\frac{1}{4}x^2 + y^2 = 1$ at $P: (\sqrt{2}, 1/\sqrt{2})$ (6 points)

(b) Using Green's theorem, evaluate the line integral $\oint_C \mathbf{F}(\mathbf{r}) \cdot d\mathbf{r}$ counterclockwise around the boundary C of the region R

where $\mathbf{F} = [x^2 e^y, y^2 e^x]$, C the rectangle with vertices $(0, 0)$, $(2, 0)$, $(2, 3)$, $(0, 3)$ (6 points)

(c) $f = \frac{1}{r}$, $r = \sqrt{x^2 + y^2 + z^2}$, find $\iint_S \nabla f \cdot \hat{n} dA$ for $S: x^2 + y^2 + z^2 = a^2$ (8 points)

(背面尚有題目)

3. (a) Find the singular points and the corresponding residues: (10 points)

$$(i) \frac{1}{z(z-1)^2} \quad (ii) \cot(z)$$

- (b) Using the residue theorem, evaluate $I = \int_{-\infty}^{\infty} \frac{x+2}{x^3+4x} dx$ (10 points)

4. (a) Define the even and odd functions, respectively. (6 points)

- (b) Is a Fourier cosine series an even or odd function?

How about a Fourier sine series? Show your arguments. (4 points)

- (c) Show that any function, $f(x)$, can be uniquely decomposed into the sum of an even function and an odd function. (10 points)

5. Consider a two-dimensional Laplace's equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \quad \text{in the domain of } 0 \leq x \leq a \text{ and } 0 \leq y \leq b$$

$$\text{with the boundary condition of } \begin{cases} u = y & \text{at } x = 0, \quad 0 \leq y \leq b \\ u = c & \text{at } x = a, \quad 0 \leq y \leq b \\ u = c & \text{at } y = 0, \quad 0 \leq x \leq a \\ \frac{\partial u}{\partial y} = 0 & \text{at } y = b, \quad 0 \leq x \leq a \end{cases}$$

Find the solution of $u(x, y)$

(20 points)