注意：考試開始鈴響前，不得翻閱試題，

並不得書寫、畫記、作答。

國立清華大學 108 學年度碩士班考試入學試題

系所班組別：**統計學研究所**

考試科目(代碼)：**機率論**(0202)

—作答注意事項—

1. 請核對答案卷（卡）上之准考證號、科目名稱是否正確。
2. 作答中如有發現試題印刷不清，得舉手請監試人員處理，但不得要求解釋題意。
3. 考生限在答案卷上標記「⬇由此開始作答」區內作答，且不可書寫姓名、准考證號或與作答無關之其他文字或符號。
4. 答案卷用盡不得要求加頁。
5. 答案卷可用任何書寫工具作答，惟為方便閱卷辨識，請儘量使用藍色或黑色書寫；答案卡限用 2B 鉛筆畫記；如畫記不清（含未依範例畫記）致光學閱讀機無法辨識答案者，其後果一律由考生自行負責。
6. 其他應考規則、違規處理及扣分方式，請自行詳閱准考證明上「**國立清華大學試場規則及違規處理辦法**」，無法因本試題封面作答注意事項中未列明而稱未知悉。

國立清華大學 108 學年度碩士班考試入學試題

系所班組別：統計學研究所碩士班 (0502)

考試科目（代碼）：機率論 (0202)

　　*請在【答案卷】作答

- 8 題填充題，共 20 個空格。每格以編號 (1) 至 (20)。
- 每格答對得 5 分，答錯不扣分。
- 答案卷中必須自行清楚標明每格之編號：(1), (2), ⋯, (20)並將答案寫在第一頁。
- 每格只需最後答案或式子，不須導證過程。

1. A medical test. A diagnostic test has a probability 0.90 of giving a positive result when applied to a person having a certain infectious disease. On the other hand, a probability 0.10 of giving a false positive when applied to a person not having that infectious disease. 10 % of the population are known to have that infectious disease.

(A) Determine the probability that the test result will be positive. ____(1)____

(B) Determine the probability that, given a positive result, the person has a disease. ____(2)____

(C) Determine the probability that the person will be misclassified. ____(3)____

2. Let (X,Y) be uniformly distributed on $\{(x,y): x^2 + y^2 \le 1\}$.

(A) Find the conditional density function of Y given X=x. ____(4)____

(B) Let $Z = \sqrt{X^2 + Y^2}$. Find the CDF for Z. ____(5)____

3.

(A) Let X ~ Poisson(λ) and Y ~ Poisson(μ). Assume that X and Y are independent. Find the distribution of X given X + Y. ____(6)____

(B) Let $X_1, \cdots, X_n \sim Exp(\beta)$, i.e., $f_{X_1}(x) = \beta e^{-\beta x}, x \ge 0$. Find the pdf of $Z = \max\{X_1, \cdots, X_n\}$. ____(7)____

(C) Let X and Y be independent random variables from an exponential distribution with mean 1/10. Find $P(X \ge 5Y)$. ____(8)____

4. Suppose that T is a continuous random variable as the lifetime of an object. Define the hazard function of T, $h_T(t)$, as $h_T(t) = \lim_{\Delta\to 0}\frac{P(t\le T\le t+\Delta|T\ge t)}{\Delta}$. Based on this definition, considering the following pdfs or cdfs:

(A) If $T\sim Exp(\beta)$, i.e., $f_T(t) = \beta e^{-\beta t}, t \ge 0$, then $h_T(t)$ can be calculated as ___(9)____.

(B) If $T\sim Weibull(\gamma,\beta)$, i.e., $f_T(t) = \frac{\gamma}{\beta}t^{\gamma-1}e^{-t^{\gamma}/\beta}, t \ge 0$, then $h_T(t)$ can be calculated as ___(10)____.

5. Let $X_1,\cdots,X_n$ are iid from $N(\theta,1)$ and let $\theta\sim N(0,1)$.

(A) Calculate $E(\theta|X_1,\cdots,X_n)$. __(11)____

(B) Calculate $Var(\theta|X_1,\cdots,X_n)$. __(12)____

6.

(A) Suppose $X_1,\cdots,X_n$ are iid with common expected value μ and variance σ^2. Let $Y_n = n^{-1}\sum_{i=1}^n X_i$. Calculate the limiting distribution of $\sqrt{n}(Y_n^2-\mu^2)$. ___(13)____

(B) Suppose $X_1,\cdots,X_n$ are iid with common density function $f(x) = 2x, 0 < x < 1$. Let $Y_n = n^{-1}\sum_{i=1}^n X_i^2$. Find a= ___(14)____ and b = ___(15)____ such that the limiting distribution of $\sqrt{n}(Y_n - a)/b$ is $N(0,1)$.

(C) Suppose $X_1,\cdots,X_n$ are iid Bernoulli random variables with $E(X_i) = p, i = 1,\cdots,n$. Let $Y_n = n^{-1}\sum_{i=1}^n X_i$. For $p \ne 1/2$, Calculate the limiting distribution of $\sqrt{n}\{Y_n(1-Y_n)\}$. __(16)____

7. I have two coins: one fair (P(head) = 0.5) and one biased (P(head) = 0.25).

(A) I pick one at random and toss it 100 times. Let X denote the number of heads in 100 tosses. Calculate E(X). ____(17)____

(B) I pick one at random and toss it until I see a head. Let Y denote the number of tosses to get a head. Calculate E(Y). ____(18)____

8. Let X and Y be two normal random variables with mean 0 and variance 1. The correlation coe cient between X and Y is ρ.

(A) Find a constant c so that X and Y −cX are independent. ____(19)____

(B) Calculate $E(X^2Y^2)$. ____(20)____