考試科目	計算機數學	所別	資訊科學	考試時間	3月19日 星期六 上下午第3節

(離散數學)

1. (10%) Please find the coefficient of X^{32} in $(1 + X^5 + X^9)^{10}$.

2. (10%) Please find the disjunctive normal form of the following function:

x y z	f(x, y, z)
1 1 1	1
1 1 0	0
1 0 1	0
1 0 0	1
0 1 1	0
0 1 0	0
0 0 1	0
0 0 0	1

3. (10%) Please draw the transition diagram of a finite-state automaton that accepts the given set of strings that start with *baa* over $\{a, b\}$.

4. (10%) Please solve the following recurrence relation:

$$a_n - 7a_{n-1} + 10a_{n-2} = 0 \qquad for\ n \geq 2.$$

5. (10%) Suppose G is an arbitrary digraph with n vertices. What is the largest possible number of distinct subgraphs with k vertices that G may have? (Treat isomorphic subgraphs as distinct. Choose G to maximize this number.)

6. (10%) Suppose that a tree T has N_1 vertices of degree 1, N_2 vertices of degree 2, N_3 vertices of degree 3, …N_k vertices of degree k. Find N_1 in terms of N_2, N_3…and N_k.

備考	試題隨卷繳交

命題委員： 048 (簽章) 94 年 3 月 4 日

考試科目	計算機數學	所別	資訊科學	考試時間	7 月 19 日 星期六 下午第 3 節

Linear Algebra

1. True or False (10%)
   (a) The eigenvalues of a real orthogonal matrix must be real.
   (b) The equation **Ax=b** has a least-squares solution only if **b** is in the column space of A.
   (c) Suppose **A** and **B** are n x n matrices, **B** is invertible, and **AB** is invertible. Then **A** must be invertible.
   (d) Hermitian matrices have only real eigenvalues.
   (e) Any subset of V (a vector space) containing just one vector must be linearly independent.
2. Matrix Decomposition (10%)
   (a) If **A** is an mxn real matrix with m>n, then **A** can be written using the singular value decomposition: $\mathbf{A}=\mathbf{UDV}^T$. State the properties of **U**,**D** and **V**. (6%)
   (b) Let **A** be the 4 × 4 matrix whose LU decomposition is given by

$$A=\begin{pmatrix}1&0&0&0\\2&1&0&0\\1&2&1&0\\2&1&2&1\end{pmatrix}\begin{pmatrix}2&2&2&2\\0&1&1&1\\0&0&2&2\\0&0&0&1\end{pmatrix}$$
, then **A**=? det(**A**)=? (4%)

3. Short proof.
   (a) Given a symmetric matrix **A** that is independent of a vector **x**. Show that

$$\frac{\partial}{\partial \mathbf{x}}[\mathbf{x}^T\mathbf{A}\mathbf{x}]=2\mathbf{A}\mathbf{x}$$
(5%) ($\mathbf{x}^T$: transpose of **x**)

   (b) Given any square matrix **B**, show that $\mathbf{K}=\mathbf{B}-\mathbf{B}^T$ is skew-symmetric. (5%)
4. Find the eigenvalues of the following matrix. Show all the details of your calculation to get full credit. (10%)

$$\begin{pmatrix}3&2&1&1&2&3\\1&2&3&4&5&6\\1&1&1&7&8&9\\0&0&0&4&0&0\\0&0&0&3&3&4\\0&0&0&5&4&3\end{pmatrix}$$

備考	試題隨卷繳交

命題委員： 049 （簽章） 年 月 日