考試科目	微積分	所別	應用數學系	考試時期	3 月 19 日 星期六	8:20~10:00

1. Let $f(x)=x^3-9x^2+15x$

   (a) Prove that $f(x)\geq 0$ for all $x\in[0,2]$ and find the absolute maximum value of $f(x)$ on $[0,2]$. (10%)

   (b) Evaluate $\lim_{n\to\infty}\left(\int_0^2[f(x)]^n\,dx\right)^{\frac{1}{n}}$ if exists. (Justify your answer.) (10%)

2. Show that $\int_0^{\pi} xf(\sin x)dx=\frac{\pi}{2}\int_0^{\pi} f(\sin x)dx$ and deduce the formula

$$\int_0^{\pi}\frac{x\sin x}{1+\cos^2 x}dx=\pi\int_0^1\frac{1}{1+x^2}dx$$ (20%)

3. Find the area of the region enclosed by the hypocycloid $x^{\frac{2}{3}}+y^{\frac{2}{3}}=a^{\frac{2}{3}}$, where $a>0$. (20%)

4. (a) State and prove the fundamental theorem of calculus. (10%)

   (b) Using (a) to compute $F'(0)$, where $F(x)=\int_{\sin x}^{x^2+1}e^{-t^2}dx$. (10%)

5. Find the volume of the largest rectangular parallelepiped that can be inscribed in the ellipsoid

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

   where a, b, and c are positive constants. (10%)

6. Let $\sum_{n=1}^{\infty}a_n$ be a convergent series, where $a_n\geq 0$ for all $n=1,2,\cdots$. Discuss the convergence of $\sum_{n=1}^{\infty}\frac{\sqrt{a_n}}{n^p}$ for $p\geq\frac{1}{2}$. (Justify your answer.) (10%)

備考	試題隨卷繳交
命題老師：	040 （簽章） 94 年 2 月 28 日