考試科目	統計學	所別	經濟（甲）	考試時間	3月18日 星期六 第四節

1. **Conditional Expectation** (20%)

   Let X and Y have a *trinominal distribution* with $n = 2$, $p_1 = \frac{1}{4}$, and $p_2 = \frac{1}{2}$.

   (a) Give the conditional expectation $E(Y \mid x)$. (7 %)

   (b) Give the conditional expectation $E(X \mid y)$. (7 %)

   (c) Give the correlation coefficient ρ between X and Y. (6 %)

2. **Bivariate Normal** (15%)

   Let X and Y have a bivariate normal distribution with $\mu_X = 70$, $\sigma_X^2 = 100$, $\mu_Y = 80$, $\sigma_Y^2 = 169$, and $\rho = \frac{5}{13}$. Find

   (a) $E(Y \mid X = 72)$. (5%)

   (b) $Var(Y \mid X = 72)$. (5%)

   (c) $Var(Y \mid X = 36)$. (5%)

3. **Change of Variable** (15%)

   Let X_1, X_2 have the joint probability density function

   $$f(x_1, x_2) = 2, \quad 0 < x_1 < x_2 < 1.$$

   Consider the transformation

   $$Y_1 = \frac{X_1}{X_2},$$

   $$Y_2 = X_2$$

   Find

   (a) the joint density function of Y_1 and Y_2 (5%),

   (b) and the marginal probability density function of Y_1 and Y_2 (5%).

   (c) Are they independent? (5%)

備考	試題隨卷繳交

命題委員： 133 （簽章）

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4. **Order Statistics** (10%)

Let $Y_1 < Y_2 < Y_3 < Y_4$ be the order statistics of a random sample X_1, X_2, X_3, X_4 from a uniform distribution with p.d.f.

$$f(x) = 1, 0 < x \leq 1$$

Determine the probability density function of Y_1 and Y_4.

5. **Sufficient Statistics** (20%)

(a) Using the factorization theorem to define *sufficient statistics.* (5%)

(b) State without proof what the **Rao-Blackwell Theorem**. Briefly discuss its significance. (9%)

(c) Let $X_1, X_2, ..., X_n$ be a random sample from a distribution with p.d.f.

$$f(x;\theta) = \theta x^{\theta-1}, \quad 0 < x < 1, \quad where \quad 0 < \theta.$$

Find the sufficient statistic Y for θ. (6%)

6. **Bayesian Estimation** (15%)
Let $X_1, X_2, .., X_n$ be a random sample from a gamma distribution with known α and $\theta = \frac{1}{\tau}$. Say τ has a prior p.d.f which is gamma with parameters α_0 and θ_0 so that the prior mean is $\alpha_0\theta_0$.

(a) Find the posterior p.d.f. of τ, given $X_1 = x_1, X_2 = x_2, ..., X_n = x_n$. (10%)

(b) Find the mean of this posterior distribution and write it as a function of the sample mean $\bar{X}$ and $\alpha_0\theta_0$. (5%)

7. **Tests** (5%)
State without proof the **Neyman-Pearson Lemma**.

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