國立中正大學

108 學年度碩士班招生考試

試 題

[第 1 節]

系所組別	經濟學系國際經濟學- 乙組
科目名稱	微積分

—作答注意事項—

※作答前請先核對「試題」、「試卷」與「准考證」之系所組別、科目名稱是否相符。

1. 預備鈴響時即可入場，但至考試開始鈴響前，不得翻閱試題，並不得書寫、畫記、作答。
2. 考試開始鈴響時，即可開始作答；考試結束鈴響畢，應即停止作答。
3. 入場後於考試開始 40 分鐘內不得離場。
4. 全部答題均須在試卷（答案卷）作答區內完成。
5. 試卷作答限用藍色或黑色筆（含鉛筆）書寫。
6. 試題須隨試卷繳還。

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科目名稱：微積分　　本科目共 1 頁　第 1 頁

系所組別：經濟學系國際經濟學-乙組

請回答下述問題（並詳述推理與計算過程）.

1. Let f be a bounded real-valued function defined on some closed interval $[a, b]$ that contains c, where $a < c < b$, please write down the (ϵ, δ)-definition for the following statements:

   (1.1) The limit of $f(x)$ as x approaches c is K. **(5pts.)**

   (1.2) $f(x)$ is continuous at $x = c$. **(10pts.)**

   (1.3) $f(x)$ is differentiable at $x = c$. **(10pts.)**

   (1.4) f is Riemann-integrable on $[a, b]$. **(5pts.)**

2. Test the convergence of the following series.

   (2.1) $\sum_{n=1}^{\infty} \frac{1}{n^{\pi-1}}$. **(5pts.)**

   (2.2) $\sum_{n=1}^{\infty} \frac{n-1}{\sqrt{n^6+1}}$. **(5pts.)**

   (2.3) $\sum_{n=5}^{\infty} \frac{e^n}{(n-4)!}$. **(5pts.)**

3. Find $\frac{dy}{dx}$ for each following equation:

   (3.1) $x^5 + y^5 = 5xy$. **(5pts.)**

   (3.2) $x^y = y^x$. **(5pts.)**

4. Evaluate the following integrals:

   (4.1) $\int_0^4 \sqrt{2x} e^{1-\sqrt{x^3}} dx$. **(5pts.)**

   (4.2) $\int_0^1 (1+x) e^{\frac{x}{2}} dx$. **(5pts.)**

   (4.3) $\int_{-\infty}^{\infty} \frac{x}{\sqrt{2}} e^{-\frac{1}{2}x^2} dx$. **(5pts.)**

5. Find the Taylor approximation of order **two** for each following function at the given point:

   (5.1) $f(x) = x + \frac{e^{-x^2}}{2}$ at $x = 0$. **(10pts.)**

   (5.2) $F(x, y) = e^{xy} \ln(x)$ at $(x, y) = (1, 1)$. **(10pts.)**

6. Find the maximal value of $f(x, y) = \sqrt{x} + \sqrt{y}$ in the following set:

$$\left\{(x, y) \in \mathbb{R}_+^2 | 2x + 2y \leq 5\right\}.$$

   Moreover, what are the corresponding values of x^* and y^* such that $f(x^*, y^*)$ attains the maximal vaule? **(10pts.)**