考試科目	微積分	所別	財政學系	考試時間	3月19日 星期日 第4節

1. Suppose that **(8 points)** (2% each)

$$f(x)=\begin{cases}\frac{1}{(x+1)^2} & \text{if } |x|>1\\ \frac{1}{4} & \text{if } |x|\le 1\end{cases}$$

(1). Is f continuous at $x=1$. (2). Is f differentiable at $x=1$.
(3). Is f continuous at $x=-1$. (4). Is f differentiable at $x=-1$.
Justify your answers.
[Note: Only "yes" or "no" as the answer will not be scored].

2. Let $f(x)=\sqrt{4-x^2}$ **(8 points)** (4% each)
(1). Find the point(s) on the graph of f at which the slope of the tangent line is equal to $-\frac{\sqrt{3}}{3}$.
(2). Find the equations(s) of the tangent line(s) of part (1).

3. The total monthly cost (in dollars) incurred by Cannon Precision Instruments Corporation for manufacturing q units of the model MP3 players is given by the function $C(q)=0.0025q^2+80q+10,000$ **(10 points)**
Show that the marginal cost curve will pass through the minimum point of the average cost curve.

4. Use the definition of the Euler number $e=\lim_{x\to\infty}(1+\frac{1}{n})^n$ to show that **(10 points)**
$\frac{d\ln x}{dx}=\frac{1}{x}, x>0.$

5. In a study, two countries' Lorentz curves was estimated as follows. **(10 points)**
Country A: $f(x)=\frac{11}{12}x^2+\frac{1}{12}x$
Country B: $g(x)=\frac{5}{6}x^2+\frac{1}{6}x$
(1).Compute the coefficient of inequality (Gini index) for each country's Lorentz curve. (7%)
(2). Which country has a more equitable income distribution? (3%)

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6. Suppose that $f(x)$ is continuous and nonnegative on an interval $[a, b]$. **(14 points)**

(1). Use the Riemann Sum to approximate the area of the region under the graph of f and bounded vertical lines $x = a$, $x = b$ and the x-axis (4%)

Now, let $f(x) = x^3$.

(2). Compute the Riemann Sum of f over the interval [0, 1], using four subintervals of equal length ($n = 4$) and choosing the representative points to be the midpoint of the subinterval. Then find the error of this approximation. (4%)

(3). Use the Trapezoidal Rule to approximate the area of the region, using $n = 4$. Then find the error of this approximation. (3%)

(4). Use the Simpson's Rule to approximate the area of the region, using $n = 4$. Then find the error of this approximation. (3%)

7. Evaluate the following integrals. **(10 points)** (5% each)

(1). $\int_1^e \frac{\ln^4 x}{x^3} dx$.

(2). $\int_{-\infty}^{\infty} xe^{-x^2} dx$.

8. **(10 points)**

Evaluate $\iint_R xe^{y^2} dA$

where R is the plane region bounded by the y-axis, the horizontal line $y = 4$, and the graph of $y = x^2$.

9. Suppose that we are given a data set containing n observations about the relation between x and y, $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$, **(10 points)**
The regression line by the Method of Least Squares is given by $y = f(x) = a + bx$. Show that the minimization of the sum of squared errors can give the ordinary least square estimators

$$\hat{b} = \frac{\sum_{i=1}^{n} x_i y_i - n\bar{x}\,\bar{y}}{\sum_{i=1}^{n} x_i^2 - n\bar{x}^2}, \quad \hat{a} = \bar{y} - \hat{b}\bar{x}, \text{ where } \bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} \text{ and } \bar{y} = \frac{\sum_{i=1}^{n} y_i}{n}.$$

10. Let $f(x) = \sqrt{1 + x^3} - x, x \geq -1$. **(10 points)**

Sketch the graph of the antiderivative F that satisfies the initial condition $F(-1) = 0$. (7%) Then estimate the value of $F(3)$ according to your sketch. (3%)

[Hint: In this case, the formula of F may not be derived by techniques of integration. Instead, you may draw the graph of f first (i.e., the direction field), and then use it to graph F proximately.]

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2.書寫時請勿超出格外，以免印製不清。