

考試科目	微積分	所別	財政系	考試時間	3月18日 星期日	第4節
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國立政治大學圖書館

1. Show that $|x|$ is continuous everywhere. (10 points)
2. Let $f(x) = ax^2 + bx + c, a > 0$. Find f^{-1} if the domain of f is restricted to (a) $x \geq -b/(2a)$, (b) $x \leq -b/(2a)$. (10 points)
3. Prove $\frac{d}{dx}[\log_b x] = \frac{1}{x \ln b}, x > 0$. (10 points)
4. Evaluate the integral $\int \frac{1}{x^3 - x} dx$. For what values of x is your result valid? (10 points)
5. Find values of $a, b, c,$ and d so that the function $f(x) = ax^3 + bx^2 + cx + d$ has a relative minimum at $(0,0)$ and a relative maximum at $(1,1)$. (10 points)
6. Evaluate the following definite integrals:
 $\int_{-\ln 3}^{\ln 3} \frac{e^x}{e^x + 4} dx$ $\int_{-1}^1 |e^x - 1| dx$ (10 points)
7. Let $w = (x_1^2 + x_2^2 + \dots + x_n^2)^k$, where $n \geq 2$. For what value of k does $\frac{\partial^2 w}{\partial x_1^2} + \frac{\partial^2 w}{\partial x_2^2} + \dots + \frac{\partial^2 w}{\partial x_n^2} = 0$ hold? (10 points)
8. Find the average value of $f(x, y) = x^2 - xy$ over the region enclosed by $y = x$ and $y = 3x - x^2$. (10 points)
9. Solve the following differential equation. Where reasonable, express the family of solutions as explicit functions of x .
 $\frac{\sqrt{1+x^2}}{1+y} \frac{dy}{dx} = -x$ (10 points)
10. Approximate $\sqrt{2}$ by applying Newton's Method to the equation $x^2 - 2 = 0$. (10 points)