

考試科目	計算機數學	所別	資訊科學	考試時間	3 月 17 日 星期六	第 3 節
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I (離散數學部分; 共 9 題 60%)

國立政治大學圖書館

1. [10%] True (mark 0) or false (mark x) questions:

- (a) If $2 + 2 = 5$ then $1 + 1 = 3$.
- (b) Let A, B, C, D be sets. Then $(A - B) - (C - D) = (A - C) - (B - D)$.
- (c) $\forall x \exists y P(x, y)$ implies $\exists x \forall y P(x, y)$.
- (d) There are languages that cannot be recognized by any machine.
- (e) If a relation is irreflexive and transitive, then it is asymmetric.
- (f) The formula $(P \vee \sim Q) \wedge (\sim P \vee Q)$ is not satisfiable.
- (g) If A is not a tautology, then $\sim A$ must be satisfiable.
- (h) If A, B and C are languages, then $A(B \cap C) = AB \cap AC$.
- (i) If A is a language, then $(A^*)^+ = A^+$.
- (j) There are context free languages that can be recognized by finite automata.

2. [3%] Which of the following formula is in conjunctive normal form and is logically equivalent to the formula: $(P \wedge S) \rightarrow (Q \wedge R)$

- (a) $\sim P \vee \sim S \vee (Q \wedge R)$ (b) $(\sim P \vee \sim S \vee Q) \wedge (\sim P \vee \sim S \vee R)$
- (c) $\sim(P \wedge S) \vee (Q \wedge R)$ (d) $(P \vee S) \wedge (\sim Q \vee \sim R)$

3. [3%] Suppose we have 4 algorithms designed to solve the same problem. If the running time of the 4 algorithms are expressed by divide-and-conquer recurrence relations as given below, then which algorithm would be asymptotically the best?

- (a) $f(n) = 10 f(n/3) + 10 n$
- (b) $f(n) = 5 f(n/2) + 6 n$
- (c) $f(n) = 9 f(n/3) + 2n^2$
- (d) $f(n) = 20 f(n/5) + 5 n^2$

4. [3%] Which of the following sets of boolean operators is not functionally complete?

- (a) { or, not } (b) { \rightarrow , 0 } (c) { xor, 1 } (d) { nand }

5. [5%] There are _____ ways in which 6 jobs can be assigned to 3 employees so that each employee is assigned at least one job and the hardest job is assigned to the best employee.

備 考 試 題 隨 卷 繳 交

命 題 委 員 :

067 (簽章) 96 年 2 月 27 日

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6. [9%] Answer the following questions about trees:

- (a) If T is a tree with 20 edges, then it has _____ vertices.
 (b) If T is a balanced full 3-ary tree of height 5, then it has at least _____ vertices.
 (c) If T is a full 4-ary tree with 82 leaves, then it has _____ internal vertices.

7. [5%] Solve the recurrence relation $a_n = 4a_{n-1} - 4a_{n-2}$ with the initial condition that $a_0 = 1$ and $a_1 = 6$. The solution is $a_n = \underline{\hspace{2cm}}$ for all $n \geq 0$.

8. [12%] Let $Q_n = (V_n, E_n)$ be a family of simple graphs where $n \geq 1$ and the set of vertices $V_n = \{a, b, c, d\}^n$ consists of all strings over the alphabet $\{a, b, c, d\}$ of length n (for instance, if $n = 2$, then $V_2 = \{aa, ab, ac, ad, ba, bb, \dots, dd\}$), and the set of edges E_n consists of all unordered pairs of members of V_n which differ in 1 position (e.g., if $n = 4$, then $aaab$ and $acab$ are adjacent since they differ only in position 2). The distance of two vertices in a graph is defined to be the number of edges of the shortest path between them and the diameter of a graph is defined to be the longest of all distances of all pairs of vertices of the graph.

- (a) There are _____ vertices in Q_n . [3%]
 (b) The degree of every vertex v in $Q_n = \underline{\hspace{2cm}}$ [3%]
 (c) There are _____ edges in Q_n . [3%]
 (d) Find all values of n under which Q_n has an Euler circuit [3%]

9. [10%] Show that a simple graph G with n vertices is connected if it has more than $(n-1)(n-2)/2$ edges.

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(線性代數)

國立政治大學圖書館

/0、(10%) 1. Please compute the QR-factorization of A.

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & -3 & 3 \\ -1 & 2 & 4 \end{bmatrix}$$

//、(10%) 2. By using orthogonal transformations, reduce the following quadratic forms to sums of squares.

(a) $2x_1^2 + x_2^2 - 4x_1x_2 - 4x_2x_3$

(b) $8x_1x_3 + 2x_1x_4 + 2x_2x_3 + 8x_2x_4$

/2、(10%) 3. Please use Gaussian elimination to solve the given system of equations.

$$x + 3y + 2z = 0$$

$$-x - 4y + 3z = -1$$

$$2x - z = 3$$

$$2x - y + 4z = 2$$

/3、(10%) 4. Please find the least-squares solution to the given system of equations.

$$x + y - z = 90$$

$$2x + y + z = 200$$

$$x + 2y + 2z = 320$$

$$3x - 2y - 4z = 10$$

$$3x + 2y - 3z = 220$$

備 考 試 題 隨 卷 繳 交

命 題 委 員 :

069 (簽章) 96 年 2 月 16 日

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