

東吳大學 108 學年度碩士班研究生招生考試試題

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系級	數學系碩士班 A 組(數學)	考試時間	100 分鐘
科目	微積分	本科總分	100 分

1. (4 points for each) Evaluate the following limits:

(a) $\lim_{y \rightarrow 2} \frac{y+2}{y^2+5y+6}$;

(b) $\lim_{h \rightarrow 0} \frac{\sqrt{4h+1}-1}{3h}$;

(c) $\lim_{x \rightarrow 9} \frac{2x-18}{x^2-81}$;

(d) $\lim_{x \rightarrow 0} \frac{\frac{1}{x-1} + \frac{1}{x+1}}{x}$;

(e) $\lim_{\theta \rightarrow 0} \sin(\theta)$;

(f) $\lim_{\theta \rightarrow 0} \frac{\sin(3\theta)}{5\theta}$;

(g) $\lim_{x \rightarrow 0} \frac{x - \sin(x)}{x^3}$;

(h) $\lim_{x \rightarrow \infty} x^{1/x}$.

2. (4 points for each) Find dy/dx :

(a) $y = x^3 - x^{-3} + 1$;

(b) $y = \frac{x}{4x+9}$;

(c) $y = (3x^2 - 1)^7$;

(d) $y = \sin(2x)^3$;

(e) $y = e^{\cos(x)}$.

3. (5 points) Find the slope of the tangent line of the circle $x^2 + y^2 = 25$ at the point $(x, y) = (3, 4)$.

4. (5 points for each) Find the absolute maximum and minimum values of the following functions:

(a) $f(x) = x^2$ on $[-2, 1]$;

(b) $f(x) = 4 - x^3$ on $[-2, 1]$;

(c) $f(x) = x^{\frac{2}{3}}$ on $[-2, 3]$.

5. (8 points) Show that the function

$$f(x, y) = \frac{2x^2y}{x^4 + y^2}$$

has no limit as $(x, y) \rightarrow (0, 0)$.

6. (4 points for each) Compute the following integrals:

(a) $\int_0^1 x^2 dx$;

(b) $\int_0^\pi \sin(x) dx$;

(c) $\int \sin(x)^3 dx$;

(d) $\int \ln(x) dx$;

(e) $\int \frac{5x-3}{x^2-2x-3} dx$.