

※ 考生請注意：本試題不可使用計算機。 請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (20%) If a n -dimension vector A has one average value \bar{A} over all vector elements, and the other n -dimension vector B has its average value \bar{B} over all vector elements.

(1) What is the $n \times n$ Correlation matrix C of A and B ? (10%)

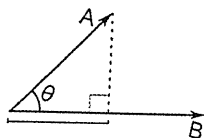
(2) What is the $n \times n$ Covariance matrix C' of A and B ? (10%)

2. (20%) If there are two vectors A and B as following figure, and the intersection angle between both vectors is θ ,

(1) What is the unit vector of A ? (5%)

(2) What is the projection length of vector A to vector B ? (Please represent the result by using θ) (5%)

(3) What is the projection length of vector A to vector B ? (Please represent the result without using θ) (10%)



3. (20%) Determine the eigenvalues (10%) and eigenvectors (10%) of the matrix:

$$A = \begin{bmatrix} 10 & 8 \\ 2 & 4 \end{bmatrix}$$

4. (20%) Given following equations:

$$a_1x_1 + a'_{12}x_2 = b_1$$

$$a_2x_1 + a'_{22}x_2 = b_2$$

(1) Please write them to be the format as $Ax=b$, where A is a 2×2 matrix, x is a 2×1 vector and b is also a 2×1 vector. (10%)

(2) Please derive the unknown x vector by using pseudo inverse approach? (Assume matrix A has no inverse format A^{-1} . That is, $x=A^{-1} b$ is incorrect. So what is the correct result of x =?) (10%)

5. (20%) For Sum of Squared Differences (SSD) format as following, please solve the unknown constant value d =?

$$\min E = \sum_{x \in R} [I(x + d) - T(x)]^2$$

by the first order Taylor series expansion: $I(x + d) \approx I(x) + d \frac{\partial I(x)}{\partial x}$ That is, starting from $\frac{\partial E}{\partial d}$:

(Note: If partially correct, you will obtain partial points)