國立雲林科技大學 108 學年度
碩士班招生考試試題

系所：電子系
科目：工程數學(3)

1. (15%) Please solve for y=y(x).

    (a) (5%) $y'' + 2y' + y = 0, y(0) = 4, y'(0) = -6$
    (b) (5%) $x^2 y'' - 5xy' + 9y = 0$
    (c) (5%) $y + 5 = \ln(y')$

2. (15%) The ODE equation: $(3x^2 y + 6xy + \dfrac{y^2}{2})dx + (3x^2 + y)dy = 0$

    (a) (5%) Verify the ODE is not exact.
    (b) (5%) Find the integrating factor $I(x,y)$.
    (c) (5%) Find the solution of the ODE.

3. (10%) Given the equation $y'' - 6y' + 9y = \dfrac{e^{3x}}{x^2}$ find the general solution.

4. (10%) If (a) $f(t) = (t + \dfrac{1}{2})^2$, Find $L[f(t)]$, (b) $F(S) = \dfrac{1}{S(S^2 + 5)}$, Find $L^{-1}[F(S)]$.

5. (15%) Consider the equations

$$\begin{vmatrix} x & + & 2y & + & & 3z & = & 4 \\ x & + & ky & + & & 4z & = & 6 \\ x & + & 2y & + & (k+2)z & & = & 6 \end{vmatrix}$$

    where k is an arbitrary constant.
    (a) (5%) Find the reduced row echelon form (rref) of the augmented matrix of this system.
    (b) (5%) For which values of the constant k does this system have infinitely many solutions? In addition, find these infinitely many solutions.
    (c) (5%) For which values of the constant k does this system have no solution?

6. (20%) Consider the vector $\mathbf{v} = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$ and a transformation $T(\mathbf{x}) = \mathbf{v} \cdot \mathbf{x}$ from $\mathcal{R}^3$ to $\mathcal{R}$ in which the notation $\cdot$ denotes the inner product.

    (a) (10%) Prove $T(\mathbf{x})$ is linear and find the transformation matrix of T.
    (b) (10%) Explain $T(\mathbf{x})$ is invertible and find its inverse transformation.

7. (15%) A data set has three points $(x, y) = (0, 6), (1, 0)$, and $(2, 0)$.
    (a) (5%) Find the closest line $y(x) = a + bx$ to these points.
    (b) (5%) Find the closest quadratic function $y(x) = c + dx + ex^2$ to these points.
    (c) (5%) Which function gives better data fitting? Explain your reason.