

考試科目	微積分	所別	財政	考試時間	3月16日 星期日	第4節
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10 points for each of the following ten questions:

- Find the value of $a + b$ for which the limit $\lim_{x \rightarrow \infty} \left(\frac{x^2 + 1}{x + 1} - ax - b \right) = -2$.
- Let $f : [0, \infty) \rightarrow \mathbb{R}$ be defined by $f(x) = \int_1^{e^x} \frac{dt}{\sqrt{1+t^2}}$.
 - Find the first derivative $f'(x)$ of $f(x)$.
 - Show that $f(x)$ has an inverse $f^{-1}(x)$ and compute $(f^{-1})'(0)$.
- Evaluate the following derivatives.
 - Let $f(x) = (1+x)^x$.
 - Let $g(x) = (x-1)(x-2)(x-3)(x-4)\cdots(x-80)$, evaluate $g'(1)$.
- Evaluate the following integrals.
 - $\int_0^1 \frac{x^2}{(1+x)^7} dx$.
 - $\int_1^{\infty} \frac{x \ln x}{(1+x^2)^2} dx$.
- Calculate the approximate value of $\sqrt[3]{1001}$ by using the total differential.
- Evaluate $\int_0^1 \int_{x^2}^1 \frac{x^3}{\sqrt{x^4 + y^2}} dy dx$.
- Find the absolute maximum and minimum values of $f(x) = x^2 e^{-x}$ in $[0, \infty)$.
- Find the derivative $\frac{dy}{dx}$ of $x^2 + y^2 = 3x^2 y - y^3$ at $x = 1$ and $y = 1$.
- Find the area of the region enclosed by the curve $y^2 = x^2(4 - x^2)$.
- Assume that $u(x, y)$ satisfies the equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$. Show that $v(x, y) = u\left(\frac{x}{x^2 + y^2}, \frac{y}{x^2 + y^2}\right)$ satisfies the equation $\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0$.

備 考 試 題 隨 卷 繳 交

命 題 委 員 :

(簽章) 97年 3 月 日

命題紙使用說明：1. 試題將用原件印製，敬請使用黑色墨水正楷書寫或打字（紅色不能製版請勿使用）。
2. 書寫時請勿超出格外，以免印製不清。
3. 試題由郵寄遞者請以掛號寄出，以免遺失而示慎重。