

國立臺灣科技大學 108 學年度碩士班招生試題

系所組別：機械工程系碩士班甲組、乙組、丙組、丁組

科目：工程數學

(總分為 100 分)

1. Solve the following ordinary differential equations:

(a) $y' = y^2 - 2xy + x^2 + 3$ (15%)

(b) $x \sin 2y dx + (x^2 \cos 2y + 3y) dy = 0$ (15%)

2. Use the Laplace transform to solve the initial problem: (15%)

$y'' + 4y' + 3y = e^t ; y(0) = 0, y'(0) = 2$

3. Find the work done by a force $\vec{F} = x^2 \vec{i} - 2yz \vec{j} + z \vec{k}$ moves a particle along the straight line from (1,1,1) to (4,4,4). (15%)4. Use Green theorem to evaluate $\oint_C \vec{F} \cdot d\vec{R}$. All curves are oriented counterclockwise.

$\vec{F} = x^2 y \vec{i} - xy^2 \vec{j}$, and C is the boundary of the region $x^2 + y^2 \leq 4, x \geq 0, y \geq 0$. (20%)

5. (a) Apply separation of variables to solve the following partial differential equation

$$\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2} \quad \text{for } 0 < x < L, t > 0.$$

Boundary conditions $u(0, t) = u(L, t) = 0$ for $t \geq 0$.

Initial condition $u(x, 0) = 10 \sin\left(\frac{\pi x}{L}\right)$ for $0 \leq x \leq L$. (10%)

(b) Determine the coefficients in the function of $u(x, t)$ based on the boundary and initial conditions. (10%)