



(You need to define and explain your notations clearly. Points are granted only if detail computations are presented.)

1. (25%) Consider the following problem:

$$\begin{aligned} \max \quad & Z = 10x_1 + 15x_2 \\ \text{s.t.} \quad & 4x_1 + 8x_2 \leq 32 \\ & x_1 - 2x_2 \geq 2 \\ & x_1, x_2 \geq 0. \end{aligned}$$

- Formulate the corresponding dual problem. (5%)
 - Compute all the complementary basic solutions of the primal and dual problems. (12%)
 - Based on (b), what is the optimal solution by applying strong duality property. (8%)
2. (25%) A transportation company has to select the daily routes from six possible routes to serve the demand points (A~G). The company has only **three** vehicles available for the transportation and aim to minimize the total cost. Demand point F has to be served at least twice a day, but the remaining demand points ($\{A, B, \dots, G\} \setminus F$) have to be served at least once a day. The information of routes and the corresponding costs is as follows:

- Route 1: A→B→C→G, \$12
- Route 2: B→D, \$6
- Route 3: C→E, \$8
- Route 4: D→E→F, \$9
- Route 5: A→B→C→D→F→G, \$15
- Route 6: E→F→G, \$10

- Formulate this problem. (8%)
- When a route is selected, a setup cost \$10 will incur. Explain how to modify the model in (a). (5%)
- If route 2 is selected, either route 5 or route 6 must be selected. Show how to modify the model in (a)? (5%)
- If route 3 is selected, route 5 and route 6 is not allowed to coexist. Show how to modify the model in (a)? (7%)



3. (20%) You are an umbrella seller. The number of umbrella you sold each day depends on the weather of that day. If it rains, you sell 100 umbrellas that day. If it does not rain, you sell 50 umbrella that day. Let us classify weather of a day as rainy (R) and not rainy (N). Let X_n be weather of day n. Assume that $\{X_n, n=1,2,\dots\}$ is a Markov chain with the following transition probability matrix. Find the expected number of umbrellas you sell in a day in steady state.

$$\begin{array}{c|cc} & R & N \\ \hline R & 2/5 & 3/5 \\ \hline N & 1/2 & 1/2 \end{array}$$

4. (30%) Consider a M/M/2/4 queue with inter-arrival time being Exp(2) and service time being Exp(1).
- Draw a transition rate diagram of this queue. (10%)
 - Find the expected number of jobs in the system (10%)
 - Find the expected time a job in the system (10%)