國立雲林科技大學 108 學年度 碩士班招生考試試題

系所：機械系

科目：工程數學(1)

1. Please find the general solution of the following ordinary differential equations, where y is a function of x :

   (a) (15%) $y'' + 4y' + 4y = e^{-2x}$

   (b) (15%) $xy' + y = xe^{x} + 3x^{2}$

2. (20%) Consider the 2nd order O.D.E.

$$y'' + 4y = f(t), \qquad y(0) = y'(0) = 1$$

y is a function of t. Please find the initial value problems for all $t \geq 0$ according to the $f(t)$ defined as follows.

$$f(t) = \begin{cases} 1 & for \quad 0 \leq t < 1 \\ 0 & for \quad t \geq 1 \end{cases}$$

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3. A line in 3-dimensionl space R^3 is represented by a position vector $\vec{p}$ in R^3 and given as $\vec{p} = t\vec{v}$,

where $t \in R$ *and* $\vec{v} \in R^3$.

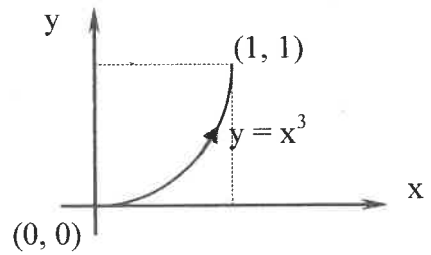
A linear transformation defined by the projection of $\vec{x}$ onto the line given above.

(a) (10%) Please define the linear transformation in vector form.

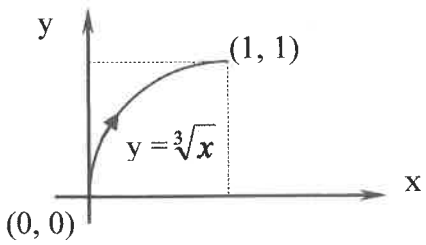
(b) (15%) Please find the eigenvalues and the corresponding eigenvectors of the linear transformation.

4. Given $\vec{F}(t) = -2y\,\vec{i} + (5y - 2x)\vec{j}$ and $\vec{r}(t) = x\,\vec{i} + y\,\vec{j}$, please compute the line integral $\int_C \vec{F} \cdot d\vec{r}$ with C as the following paths.

(a) (8%)



(b) (8%)



(c) (9%) Please show the reason why they have the same results.

(Hint: **Try to find the potential function** $\phi(x,y)$ which $\vec{F}$ is derived from.)