逢甲大學108學年度碩士班考試入學試題

編號：07　　科目代碼：205

科目	微積分	適用系所	統計學系統計與精算碩士班應用統計暨計量財務組、精算組	時間	90分鐘

※請務必在答案卷作答區內作答。　　共 一 頁 第 一 頁

1. (5%+7%+8%) (a) $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x}$;

   (b) Use the result (a) to compute $\lim_{x\to\infty}\left(\frac{x+\ln 2}{x-\ln 2}\right)^{x}$;

   (c) Does the series $\sum_{x=1}^{\infty}\left(1+\frac{1}{x}\right)^{x^2}$ converge or diverge? Explain it.

2. (10%) Let $M(t_1,t_2)=\left[p_1e^{t_1}+p_2e^{t_2}+(1-p_1-p_2)\right]^n$ and $\psi(t_1,t_2)=\ln M(t_1,t_2)$, where p_1, p_2, and n are real numbers. Calculate $\frac{\partial^2\psi(t_1,t_2)}{\partial t_1\partial t_2}$ at $(t_1,t_2)=(0,0)$.

3. (10%) Let $F(x)=\int_{-\sqrt{x}}^{\sqrt{x}}\frac{1}{\sqrt{2\pi}}e^{\frac{-y^2}{2}}dy$. Find $\frac{dF(x)}{dx}$.

4. (10%) Find the slope of the curve $x^4-5x^3y+2y^8=1$ at $(x,y)=(2,6)$.

5. (10%+10%+10%) Calculate the following integrals:

   (a) $\int_1^2 x^{2019}\,dx$

   (b) $\int_0^{13}\frac{1}{\sqrt[3]{(1+2x)^2}}dx$

   (c) $\int_1^2\frac{\ln x}{x^2}dx$

6. (10%+10%) Calculate the following double integrals:

   (a) $\int_0^4\int_{\sqrt{x}}^2 x\cdot(\cos(y^5))\,dydx$

   (b) $\iint_A cos(x^2+y^2)dxdy$, where $A=\left\{(x,y)|\ x^2+y^2\le\frac{\pi}{2},x\ge 0,y\ge 0\right\}$.