

元智大學 108 學年度 碩士班 招生試題卷

系(所)別：機械工程學系碩士班 組別：不分組

科目：工程數學

用紙第 / 頁共 2 頁

● 不可使用電子計算機

1. Solve the initial value problem (16%)

$$y' = -\frac{y}{x} \quad \text{with} \quad y(1) = 1$$

2. Using the method of Laplace Transformation to solve the initial value problem of $y(t)$ (17%)

$$y'' + 4y' + 3y = e^t \quad \text{with} \quad y(0) = 0, \quad \left. \frac{dy}{dt} \right|_{t=0} = 2$$

3. Find the directional derivative of $f = \arctan \frac{y}{x}$ at $P: (1, 1, 0)$ in the direction of $\vec{a} = [1, -2, 2]$. (10 %)

4. For the matrix $A = \begin{bmatrix} 5 & -1 & 0 \\ -1 & 5 & 0 \\ 0 & 0 & 4 \end{bmatrix}$,

- (1) Find the eigenvalues of A . (5 %)
(2) Find the inverse of A^2 . (8 %)

5. Using Green's theorem, evaluate the line integral $\oint_C \vec{F}(\vec{r}) \cdot d\vec{r}$ counterclockwise around the boundary C of the region R , where $\vec{F} = \left[\frac{e^y}{x}, e^y \ln x + 2x \right]$, $R: 1 + x^4 \leq y \leq 2$. (10 %)

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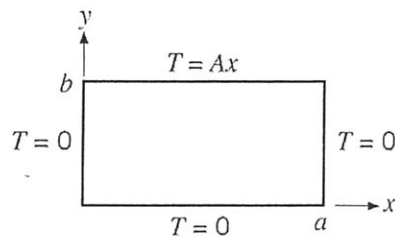
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6. A two-dimensional rectangular plate is subjected to the boundary conditions shown as below. Derive an expression for the steady state temperature distributions $T(x,y)$ with solving the heat conduction equation. (17%)



The heat conduction equation is :

$$\frac{\partial^2 T(x,y)}{\partial x^2} + \frac{\partial^2 T(x,y)}{\partial y^2} = 0, \text{ and boundary conditions as above.}$$

Please find the solution in sin, cos, sinh, cosh series functions by the method of separation variables.

7. There is periodic square wave with analytic represented as $f(x)$ function

$$f(x) = \begin{cases} -k & \text{when } -\pi < x < 0 \\ k & \text{when } 0 < x < \pi \end{cases}$$

and $f(x+2\pi) = f(x)$

Please find the Fourier coefficient of a_n, b_n and their series functions to present the $f(x)$ functions. (17%)