●不可使用電子計算機

1. Prove whether the following series coverage or diverge, show your work.

   a. (5 points) $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1}{n+1}\right)$

   b. (5 points) $\displaystyle\sum_{n=1}^{\infty}\frac{\sin(n)}{n}$

   c. (10 points) Find the number of terms is needed for the error of estimated sum

   of $\displaystyle\sum_{n=1}^{\infty}\left(\frac{5}{2n^{5}}\right)$ is less than 0.00005.

2. (10 points each) evaluate the integrals

   a. $\displaystyle\int\frac{dx}{\sqrt{9x^{2}+18x+10}}$

   b. $\displaystyle\int\tan^{4}x\sec^{3}x\,dx$

3. (15 points) Find the gradient and Hessian for the following functions
   $$f(x,\,y)=4x^{2}y+2x^{2}-6e^{y}+2y$$

4. (15 points) $\displaystyle\iint_{R}xy\,dA$ where R is the region in the first quarter and bounded by $y=x^{2}$

   and $y=3x$. Evaluate the integral.

5. (10 points) Use the inverse function of ln$x$ to show $\dfrac{d\ln x}{dx}=\dfrac{1}{x}$

6. (10 points) Use the squeeze theory to show $\lim_{x\to0}\dfrac{\sin(x)}{x}=1$.

7. (10 points) Find the volume of the region bounded by the $x=(y-1)^{2}$, $x-y=1$ and
   rotated about the $x=1$.

Good Luck!

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