國立高雄大學 108 學年度研究所碩士班招生考試試題

科目：統計學　　系所：金融管理學系　　是否使用計算機：是

考試時間：100 分鐘　　本科原始成績：100 分

1. (5%) The following table shows the percentage of individuals in each age group who use an online tax program to prepare their federal income tax return (CompleteTax website, November 9, 2012). Suppose a follow-up study consisting of personal interviews is to be conducted to determine the most important factors in selecting a method for filing taxes. How many 18–34-year-olds must be sampled to find an expected number of at least 25 who use an online tax program to prepare their federal income tax return?

Age	Online tax program (%)
18-34	16
35-44	12
45-54	10
55-64	8
65+	2

2. (5%) Management proposed the following regression model to predict sales at a fast-food outlet:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \varepsilon,$$

where x_1= number of competitors within one mile, x_2 = population within one mile (1000s), x_3 = 1 if drive-up window present, and 0 otherwise, and y = sales ($1000s). The following estimated regression equation was developed after 20 outlets were surveyed:

$$\hat{y} = 10.1 - 4.2x_1 + 6.8x_2 + 15.3x_3.$$

Please predict sales for a store with two competitors, a population of 8000 within one mile, and no drive-up window.

3. Data on advertising expenditures and revenue (in thousands of dollars) for the Four Seasons Restaurant follow.

Advertising expenditures	Revenue
1	19
2	32
4	44
6	40
10	52
14	53
20	54

(1) (5%) Let *x* equal advertising expenditures and *y* equal revenue. Use the method of least squares to develop a straight line approximation of the relationship between the two

variables.

(2) (5%) Test whether revenue and advertising expenditures are related at a 0.05 level of significance. *F* value (1 degree of freedom numerator and 5 denominator) at a 0.10 level of significance is equal to 4.06, at a 0.05 level of significance is equal to 6.61, at a 0.025 level of significance is equal to 10.01.

(3) (5%) Prepare a residual plot of $\mathbf{y} - \hat{y}$ versus $\hat{y}$. Use the result from part (1) to obtain the values of $\hat{y}$.

(4) (5%) What conclusions can you draw from residual analysis? Should this model be used, or should we look for a better one?

4. (5%) The price per share of stock for a sample of 25 companies was recorded at the beginning of 2012 and then again at the end of the 1st quarter of 2012 (the *Wall Street Journal*, April 2, 2012). How stocks perform during the 1st quarter is an indicator of what is ahead for the stock market and the economy. Use the sample data in the following.

Company	End of 1st Quarter	Beginning of Year	Company	End of 1st Quarter	Beginning of Year
Bank of New York	24.13	19.91	Home Depot	50.31	42.04
Kraft Foods	38.01	37.36	JP Morgan Chase	45.98	33.25
Dupont	52.90	45.78	Procter & Gamble	67.21	66.71
Consolidated Edison	58.42	62.03	Verizon	38.23	40.12
Johnson & Johnson	65.96	65.58	Devon Energy	71.12	62.00
Union Pacific	107.48	105.94	Lilly	40.27	41.56
Comcast	30.01	23.71	Microsoft	32.26	25.96
Applied Materials	12.45	10.71	Coco Cola	74.01	69.97
Pfiser	43.41	41.22	Qualcomm	68.06	54.70
General Electric	20.07	17.91	Exxon Mobil	86.73	84.76
AT&T	31.23	30.24	Pacific Gas & Electric	22.65	21.64
Cisco Systems	21.15	18.08	Oracle Systems	29.16	25.58
			Chevron	107.21	106.40

What is the 95% confidence interval estimate of the population mean change in the price per share of stock during the first quarter? (With *df* = 24, $t_{0.025}$=2.064, $t_{0.05}$=1.711)

5. (5%) Investors commonly use the standard deviation of the monthly percentage return for a mutual fund as a measure of the risk for the fund; in such cases, a fund that has a larger standard deviation is considered more risky than a fund with a lower standard deviation. The standard deviation for the American Century Equity Growth fund and the standard deviation for the Fidelity Growth Discovery fund were recently reported to be 15.0% and 18.9%, respectively. Assume that each of these standard deviations is based on a sample of 60 months of returns. Do the sample results support the conclusion that the Fidelity Fund has a larger population variance

than the American Century Fund? Which fund is more risky? (α = 0.05)

$F_{(0.05,59,59)}$=1.54
$F_{(0.95,59,59)}$=0.65

6. In a regression analysis involving 27 observations, the following estimated regression equation was developed:

$$\hat{y} = 25.2 + 5.5x_1.$$

For this estimated regression equation SST = 1550 and SSE = 520.

(1) (5%) At α = 0.05, test whether x_1 is significant. Suppose that variables x_2 and x_3 are added to the model and the following regression equation is obtained:

$$\hat{y} = 16.3 + 2.3x_1 + 12.1x_2 - 5.8x_3.$$

For this estimated regression equation SST = 1550 and SSE = 100.

(2) (5%) Use an F test and α = 0.05 level of significance to determine whether x_2 and x_3 contribute significantly to the model.

$F_{(0.05,1,25)}$=4.24　$F_{(0.05,2,23)}$=3.42
$F_{(0.95,1,25)}$=0.004　$F_{(0.95,2,23)}$=0.051
$F_{(0.025,1,25)}$=5.69　$F_{(0.025,2,23)}$=4.35
$F_{(0.975,1,25)}$=0.001　$F_{(0.975,2,23)}$=0.025

7. Let X and Y be two random variables defined on the same probability space. Please show that:

(1) (5%) $Var(X) = \mathrm{E}\{X^2\} - (\mathrm{E}\{X\})^2$, where $Var(\square)$ and $\mathrm{E}\{\square\}$ denote the variance and the expectation of a random variable, respectively.

(2) (5%) $Cov(X,Y) = \mathrm{E}\{XY\} - \mathrm{E}\{X\}\mathrm{E}\{Y\}$, where $Cov(\square)$ denotes the covariance of two random variables.

8. Let X be uniformly distributed on $[0,1]$ and Y be normally distributed with mean 0 and variance 1. If a and $b \neq 0$ are real numbers, please find the probability density functions of

the following random variables:

(1) (5%) $a+bX$.

(2) (5%) $a+bY$.

(3) (7%) e^{a+bY}.

9. Let $\{W_0, W_1, W_2, ...\}$ be a group of random variables which satisfies the following three conditions:

i. $W_0 = 0$.

ii. $W_i - W_j$ is normally distributed with mean 0 and variance $i-j$, where $0 \le j < i$.

iii. Given $0 \le k < j < i$, $W_i - W_j$ and $W_j - W_k$ are independent.

Please answer the following questions:

(1) (10%) What is the covariance of W_i and W_j, where $0 \le j < i$?

(2) (13%) Given a positive real number K. Please show that:

$$\mathrm{E}\left\{\max\left\{e^{W_i} - K, 0\right\}\right\} = e^{-\frac{i}{2}} N\left(\frac{\ln K + i}{\sqrt{i}}\right) - KN\left(\frac{\ln K}{\sqrt{i}}\right),$$

where $\max\left\{e^{W_i} - K, 0\right\}$ denotes the maximum between $e^{W_i} - K$ and 0, as well as

$$N(d) = \int_{-\infty}^{d} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx.$$

Table 1: *t*-Distribution Critical Values

The entries in the table below are the critical values $t_{n,p}$, where n represents the number of degrees of freedom and p is the upper tail probability. That is, if T has the t-distribution with n degrees of freedom, then the value in the table represents the number $t_{n,p}$ such that $P\left(T > t_{n,p}\right) = p$.

	Upper Tail Probability p									
d.f.	0.20	0.15	0.10	0.05	0.025	0.01	0.005	0.0025	0.001	0.0005
1	1.376	1.963	3.078	6.314	12.706	31.821	63.657	127.321	318.309	636.619
2	1.061	1.386	1.886	2.920	4.303	6.965	9.925	14.089	22.327	31.599
3	0.978	1.250	1.638	2.353	3.182	4.541	5.841	7.453	10.215	12.924
4	0.941	1.190	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	0.920	1.156	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	0.906	1.134	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	0.889	1.108	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	0.883	1.100	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	0.879	1.093	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	0.876	1.088	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.768
24	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690
28	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659
30	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
35	0.852	1.052	1.306	1.690	2.030	2.438	2.724	2.996	3.340	3.591
40	0.851	1.050	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
45	0.850	1.049	1.301	1.679	2.014	2.412	2.690	2.952	3.281	3.520
50	0.849	1.047	1.299	1.676	2.009	2.403	2.678	2.937	3.261	3.496
55	0.848	1.046	1.297	1.673	2.004	2.396	2.668	2.925	3.245	3.476
60	0.848	1.045	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
65	0.847	1.045	1.295	1.669	1.997	2.385	2.654	2.906	3.220	3.447
70	0.847	1.044	1.294	1.667	1.994	2.381	2.648	2.899	3.211	3.435
75	0.846	1.044	1.293	1.665	1.992	2.377	2.643	2.892	3.202	3.425
80	0.846	1.043	1.292	1.664	1.990	2.374	2.639	2.887	3.195	3.416
85	0.846	1.043	1.292	1.663	1.988	2.371	2.635	2.882	3.189	3.409
90	0.846	1.042	1.291	1.662	1.987	2.368	2.632	2.878	3.183	3.402
95	0.845	1.042	1.291	1.661	1.985	2.366	2.629	2.874	3.178	3.396
100	0.845	1.042	1.290	1.660	1.984	2.364	2.626	2.871	3.174	3.390
150	0.844	1.040	1.287	1.655	1.976	2.351	2.609	2.849	3.145	3.357
250	0.843	1.039	1.285	1.651	1.969	2.341	2.596	2.832	3.123	3.330
1000	0.842	1.037	1.282	1.646	1.962	2.330	2.581	2.813	3.098	3.300
∞	0.842	1.036	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291