國立高雄大學 108 學年度研究所碩士班招生考試試題

科目：離散數學與資料結構　　系所：資訊工程學系　　是否使用計算機：否

考試時間：100 分鐘　　本科原始成績：100 分

1. (a) (5%) In how many ways can 10 people be arranged into 5 groups labeled A, B, C, D, and E, such that each group has exactly 2 persons?
(b) (5%) In how many ways can 10 people be arranged into 5 groups without group labeling such that each group has exactly 2 persons?

2. (10%) Show that $\sum_{i=1}^{n}(i)(i!) = (n+1)! - 1$ for all $n \geq 1$ by the Principle of Mathematical Induction.

3. For $m \geq n$, the Stirling number of the second kind, $S(m,n)$, is $\frac{1}{n!}\sum_{k=0}^{n}(-1)^k\binom{n}{n-k}(n-k)^m$.

$S(m,n)$ is equal to the number of ways in which it is possible to distribute the m distinct objects into n identical containers with no container left empty. Show that

(a) (5%) $n! = \frac{1}{n!}\sum_{k=0}^{n}(-1)^k\binom{n}{n-k}(n-k)^m$, for positive integer n.

(b) (5%) $S(m+1,n) = S(m,n-1) + n\,S(m,n),\ 1 < n \leq m$.

4. There are 5 men, M1, M2, M3, M4, and M5, and 5 women, W1, W2, W3, W4, and W5, to be matched into 5 pairs (one man matched to one woman). In how many ways can they be matched such that
(a) (5%) Mi is **not** matched to Wi, $1 \leq i \leq 5$?
(b) (5%) M1 is **not** matched to W1 or W4, M2 is **not** matched to W2 or W5, M3 is **not** matched to W1 or W4, M4 is **not** matched to W2, W3, or W5, and M5 is **not** matched to W2 or W5?

5. There are 10 red, 10 green, 10 white, and 10 black balls. John wants to select 10 balls from these 40 balls such that he has any numbers of red or green balls, even number of white balls, and odd number of black balls.
(a) (5%) Explain why the generating function for the number of ways John selects 10 balls is $f(x) = (1+x+x^2+\cdots+x^{10})^2(1+x^2+x^4+\cdots+x^{10})(x+x^3+x^5+\cdots+x^9)$?
(b) (5%) Find the number of ways John select 10 balls.

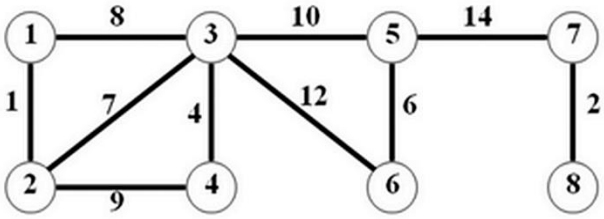
For the following three questions, please choose the best answer to the question.

6. Given that W=10, X=5, Y=5, Z=3, what value the postfix expression "WX/YZ-+X*" is? (3%)
(a) 7
(b) 5

(c) 20
(d) 10

7. Assuming that `z` is an array and `zPtr` is a pointer to that array, what expression refers to the address of the sixth element? (3%)
(a) `*(zPtr+5)`
(b) `zPtr[5]`
(c) `*(z+5)`
(d) `&z[5]`

8. In the following undirected graph each number on the edges represents the edge cost, and each number in the circles represents the vertex index. What is the cost of a minimum cost spanning tree? (3%)



(a) 42
(b) 44
(c) 45
(d) 75

9. Given the sentence “The quick brown foxes jump over the lazy dogs”:
a) Build the raw, unbalanced binary search tree using this sentence. (Hint: A string begins with an uppercase letter is the smallest) (5%)
b) Build an AVL tree using this sentence. (10%)

10. For the polynomial below, please show how to use singly linked list to represent it. (3%)

$$A(x) = 7x^{12} - 5x^4 + 6$$

11. In applications of search engine for webpages, an important question is how to construct an index. An index shall consist of data structures to assist the search for the webpages that contain particular keywords.

A simplified example is as follows:

Given three webpages:

Webpage 1: "Mary is slower than Andy, but Mary is quicker than Rick."
Webpage 2: "Andy is also good at playing basketball."
Webpage 3: "Andy is also quicker than Mike."

Given a collection of one billion webpages, in which each webpage contains about 100 words, and there are totally 100000 different words in the collection.

a) Please discuss what data structure you would suggest to determine whether a particular word exists in any webpages in the collection. Also analyze the time complexity for your approach. (10%)
b) Since there are many new webpages generated every day, and these new webpages have to be indexed, too, please also discuss what data structure you would use to determine whether a particular word exists in any webpages in the new collection. (5%)

12. Given two sorted arrays A and B (i.e., elements in the arrays are sorted) below:

Array A:

3	6	7	151	280	587	…	11075	15077	100985

Array B:

2	3	116	280	311	632	…	100985	200001	208000

In the array representation above, "…" means that there are many other sorted elements there. Please provide an algorithm to find the common elements in the two arrays and discuss the time complexity for your algorithm. (8%)