考試科目	資料結構與演算法	所別	資科系	考試時間	3月14日 星期六 第1節

可用中文或英文回答

1. (20%) True or False (Please write T or F as an answer to each of the statement)

(1) Given two sorted linked list, it is possible to merge them into one sorted linked list in linear time.

(2) Hashing can be used to implement the key insertion and finding in constant average time.

(3) Stack is helpful to improve the execution time of Dijkstra's shortest path algorithm.

(4) Dynamic programming, like the divide and conquer method, solves problems by combining the solutions to subproblems.

(5) A topological ordering is not possible if the graph is cyclic.

(6) Greedy method always leads to an optimal solution.

(7) The Floyd's all pair shortest path algorithm is a divide and conquer algorithm.

(8) If we have proved that the lower bound of an NP-complete problem is polynomial, then we have proved that NP = P .

(9) If an NP-complete problem X is polynomial reducible to a problem Y, then Y is an NP-complete problem.

(10) Every NP-complete problem must be a NP-hard problem.

2. (20%) Single selection

(1) A logical expression contains three types of operators, namely, **in order of precedence, ~ (not), ∨ (or), ∧ (and)**. Which is the postfix expression of the logical expression

G∧ (A∨B)∨ (C ∨ (F∨~D∧E))

(a) GAB∨CFD~∨E∧∨∨∧

(b) GAB∨∧CFD~E∨∨∨∧

(c) GAB∨∧CFD~E∧∨∨∨

(d) GAB∨CF~D∨E∧∨∨∧

(2) Which of the following data, inserted in the input order, will produce a complete binary search tree ?

(a) (Bill, Grace, James, John, Lily, Mary)

(b) (John, Mary, Grace, Bill, Lily, James)

(c) (Mary, Lily, John, James, Grace, Bill)

(d) (James, Grace, John, Bill, Lily, Mary)

(e) (Grace, Bill, James, John, Mary, Lily)

備考	試題隨卷繳交
命題委員：	（簽章）

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(3) Which of the following is a heap ?

(a)

1	2	3	4	5	6	7	8	9	10	11	12
50	30	46		18	25	42	60				

(b)

1	2	3	4	5	6	7	8	9	10	11	12
18	25	42	40	46	30	50					

(c)

1	2	3	4	5	6	7	8	9	10	11	12
1	2	3		5	6	7			10	11	12

(d)

1	2	3	4	5	6	7	8	9	10	11	12
18	25	23	40	46	42	53	50				

(4) Which of the following sorting algorithm takes the least number of comparisons for sorting of the following sequence of data (6, 16, 66, 78, 95, 100, 180, 229) ?

(a) selection sort

(b) quick sort

(c) insertion sort

(d) merge sort

(e) heap sort

(5) Which of the following formula is the worst case time complexity in terms of comparison operations for quick sort of n records ?

(a) $T(n)=2T(n/2)+cn$

(b) $T(n)=T(n/2)+cn$

(c) $T(n)=2T(n/2)+n^2$

(d) $T(n)=T(n-1)+cn$

(6) Which of the following formula is the worst case time complexity in terms of comparison operations for merge sort of n records ?

(a) $T(n)=2T(n/2)+cn$

(b) $T(n)=T(n/2)+cn$

(c) $T(n)=2T(n/2)+n^2$

(d) $T(n)=T(n-1)+cn$

(7) Which of the following is true for the minimum spanning tree of a graph with n vertices?

(a) The minimum spanning tree is cyclic.

(b) Prim's algorithm is a greedy algorithm while Kruskal's algorithm is not.

(c) Prim's algorithm starts from any vertices in the graph.

(d) The minimum spanning tree of this graph consists of $n+1$ edges

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(8) Which of the following is **FALSE** concerning a graph with v vertices and e edges?

(a) There exists $O(e+v)$ algorithm for single source shortest path of acyclic graph.

(b) There exist algorithms in which finding the shortest path from the source to another vertex is any faster (by more than a constant factor) than finding the shortest paths from the source to all the other vertices.

(c) In critical path analysis, the longest path is the critical path.

(d) It is possible to depth first traversal a graph in linear time.

(9) The worst case time complexity of finding the second minimum key in an n-key heap is

(a) $O(1)$ (b) $O(\log n)$ (c) $O(n)$ (d) $O(n\log n)$ (e) $O(n^2)$ (f) $O(n^2\log n)$

(10) Which of the following is **NOT** an NP-complete problem ?

(a) Traveling salesman problem.

(b) Knapsack problem

(c) Closest pair problem

(d) Clique problem

(e) Satisfiability problem

3. (10%) Please show the result of sorting 56, 6, 15, 100, 51, 38, 82 using radix sort with 7 buckets. The result of each pass must be listed.

4. (10%)

(1) Figure 1 shows the array implementation for an AVL tree T. Please produce the result after inserting 75, 99 and 32 successively into the AVL tree T.

(2) Please give the time complexity of insertion a key into an n-node AVL tree

(3) Please give the time complexity of a single rotation, LL, in an n-node AVL tree.

(4) Please give the time complexity of a double rotation, LR, in an n-node AVL tree.

(5) Please give the time complexity of finding the maximum in an n-node AVL tree.

50	29	76	12	35	63	88					60	70		95
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

Figure 1: An AVL tree T.

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5. (10 %) Given the following algorithm, please analyze and give
   (1) the best time complexity of the comparison operation.
   (2) the best time complexity of the exchange operation.
   (3) the worst time complexity of the comparison operation.
   (4) the worst time complexity of the exchange operation.

```
void X-Algorithm(int A[], int N)
{  int j, p, m;
   for (p=0; p < N-2; p++)
       {   m=p;
           for (j=p+1; j < N-1 ; j++)
               if (A[j] < A[m])
                   m=j;
           exchange (A[p], A[m]); };
}
```

6. (10%) There is a town with n citizens. It is known that some pairs of people are friends. According to the famous saying that "The friends of my friends are my friends, too" it follows that if A and B are friends and B and C are friends then A and C are friends, too.
   (1) Please design and illustrate an $O(n^3)$ algorithm, using the example matrix X shown in Figure 2, to determine whether two citizens are friends for each pair of people. In this matrix X, there are 7 citizens. If two citizens i and j are friends, then the cells x_{ij} and x_{ji} are denoted as '1'. Otherwise, the cells are denoted as '0'.
   (2) Please design and illustrate an $O(n)$ algorithm, using the example matrix X, to count how many people there are in the largest group of friends.

	1	2	3	4	5	6	7
1	0	0	0	0	1	1	0
2	0	0	1	0	0	0	0
3	0	1	0	0	0	0	0
4	0	0	0	0	1	0	1
5	1	0	0	1	0	0	0
6	1	0	0	0	0	0	0
7	0	0	0	1	0	0	0

Figure 2: An example matrix $\boldsymbol{X}$

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7. (10%) A sequence $x_1, x_2, \ldots x_n$ is said to be cyclically sorted if the smallest number in the sequence is x_i for some unknown i, and the sequence $x_i, x_{i+1}, \ldots x_n\ x_1, \ldots, x_{i-1}$ is sorted in increasing order. Please design and illustrate an O(log n) algorithm to find the position of the minimal element in a cyclic sorted sequence of n elements by using the example sequence 45, 52, 66, 72, 3, 29, 38.

8. (10%) Given the following structures and type specification of AVL trees,
   (1) please write the routine in C Language to count the number of nodes. The prototype of the function is defined as int count(AVLTree root).
   (2) please write the routine in C Language to list the element of each node in descending order. The prototype of the function is defined as void descending(AVLTree root).

```
typedef int DATA;
struct node {
    DATA          element;
    struct node   *left;
    struct node   *right;
    int           height;
};
typedef  struct node   AVLNODE;
typedef  AVLNODE    *AVLTree;
```

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