考試科目	計算機數學	所別	資訊科學系	考試時間	3月14日 星期六 第3節

1. (15%) (a). (5%) Find the least positive integer x satisfying the congruence:

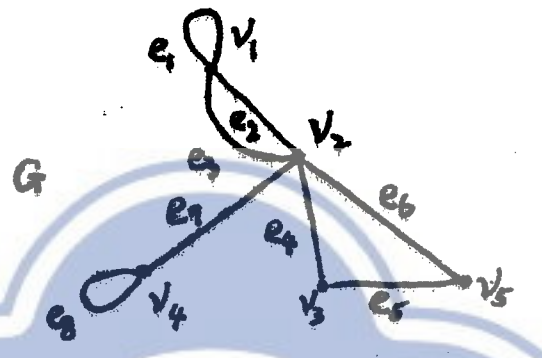
$$x \equiv 325^{1329} \bmod 11.$$

(b). (10%) Find the least positive integer x satisfying the congruence:

$$531x \equiv 1 \bmod 1769.$$

2. (10%) (a). (4%) How many vertices and how many edges are there in the complete bipartite graphs $K_{m,n}$.

(b). (6%) Present the pseudograph G using an incidence matrix.



3. (5%) Prove that there are infinitely many primes.

4. (10%) Find the state diagram for the nondeterministic finite-state automaton with the following state table. The final states are s_2 and s_3.

State Table

State	f: Input 0	f: Input 1
s_0	s_0, s_1	s_3
s_1	s_0	s_1, s_3
s_2		s_0, s_2
s_3	s_0, s_1, s_2	s_1

5. (10%) $x_1+x_2+x_3+x_4+x_5+x_6+x_7=37$, $x_1+x_2+x_3=6$ and $x_1, x_2, x_3>0$. Find the number of nonnegative integer solutions.

備考	試題隨卷繳交

命題委員： （簽章） 日

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6. (10%)Let v, k and λ be positive integers such that $v>k\geqq 2$. A (v, k, λ)-balanced incomplete block design (abbreviate to (v, k, λ)-BIBD) is a pair (X,A) such that the following properties are satisfied:

(1). X is a set of v elements called points,

(2). A is a collection of subsets of X called blocks,

(3). each block contains exactly k points, and

(4). every pair of distinct points is contained in exactly λ blocks.

For example, a pair (X,A) with

$X=\{1,2,3,4,5,6,7\}$ and $A=\{123,145,167,246,257,347,356\}$ is a $(7,3,1)$-BIBD.

(a). (5%) In how many blocks does each point occur in a (v, k, λ)-BIBD?

(b). (5%) How many blocks does a (v, k, λ)-BIBD has?

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7. (10%) Solve the following system using its augmented matrix M:

$$x + 2y - z = 3$$

$$x + 3y + z = 5$$

$$3x + 8y + 4z = 17$$

8. (10%)

Let $A = \begin{bmatrix} 1 & 2 & 1 & 2 & 3 & 1 \\ 2 & 4 & 3 & 7 & 7 & 4 \\ 1 & 2 & 2 & 5 & 5 & 6 \\ 3 & 6 & 6 & 15 & 14 & 15 \end{bmatrix}$.

(a) Find rank(M_k), for k = 1, 2, …, 6, where M_k is the submatrix of A consisting of the first k columns $C_1, C_2, \ldots, C_k$ of A.

(b) Which columns C_{k+1} are linear combinations of proceeding columns $C_1, C_2, \ldots, C_k$?

9. (10%) Consider the following subspaces of $\mathbf{R}^5$:

$U = \text{span}(u_1, u_2, u_3) = \text{span}\{(1, 3, -2, 2, 3), (1, 4, -3, 4, 2), (2, 3, -1, -2, 9)\}$

$W = \text{span}(w_1, w_2, w_3) = \text{span}\{(1, 3, 0, 2, 1), (1, 5, -6, 6, 3), (2, 5, 3, 2, 1)\}$

Find a basis and the dimension of $U + W$.

10. (10%)

Let $A = \begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$.

(a) Find all eigenvalues and corresponding eigenvectors.

(b) Find a nonsingular matrix P such that $D = P^{-1}AP$ is diagonal, and P^{-1}.

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