

# 國立中正大學

## 108 學年度碩士班招生考試

### 試題

#### [第 1 節]

系所組別	電機工程學系-信號與媒體通訊組
	通訊工程學系-通訊甲組
科目名稱	通訊原理

#### —作答注意事項—

※作答前請先核對「試題」、「試卷」與「准考證」之系所組別、科目名稱是否相符。

1. 預備鈴響時即可入場，但至考試開始鈴響前，不得翻閱試題，並不得書寫、畫記、作答。
2. 考試開始鈴響時，即可開始作答；考試結束鈴響畢，應即停止作答。
3. 入場後於考試開始 40 分鐘內不得離場。
4. 全部答題均須在試卷（答案卷）作答區內完成。
5. 試卷作答限用藍色或黑色筆（含鉛筆）書寫。
6. 試題須隨試卷繳還。

**I. Short Questions**

Answer the questions below by providing the most appropriate choice. Write down the correct answer on your answer sheet. No explanations will be considered in grading this portion of the exam. Each correct answer is worth 5 points (5%).

- To combat the effects of aliasing in practice, which of the following methods is correct?
  - Use a high-pass filter to attenuate the in-band noise.
  - The signal is sampled at a rate slightly lower than the Nyquist rate.
  - Reduce the signal amplitude so that aliasing effect can be reduced.
  - Use a low-pass filter to attenuate those high frequency components of the signal before sampling.
  - None of the above.
- A PAM system produces the signal  $s(t) = \sum_{n=-\infty}^{\infty} m(nT_s)h(t - nT_s)$ , where  $T_s$  is the sampling period and  $m(t)$  is the message signal. Let  $H(f)$  be the Fourier transform of  $h(t)$  and  $M(f)$  be the Fourier transform of  $m(t)$ . Determine the Fourier transform of  $s(t)$ .
  - $S(f) = \frac{1}{T_s} \sum_{k=-\infty}^{\infty} H\left(f - \frac{k}{T_s}\right) M(f)$
  - $S(f) = \frac{1}{T_s} \sum_{k=-\infty}^{\infty} M\left(f - \frac{k}{T_s}\right) \otimes H(f)$ , where  $\otimes$  denotes the convolution operator.
  - $S(f) = \frac{1}{T_s} \sum_{k=-\infty}^{\infty} M\left(f - \frac{k}{T_s}\right) H(f)$
  - $S(f) = \frac{1}{T_s} \sum_{k=-\infty}^{\infty} M\left(f - \frac{k}{T_s}\right) H(f) \exp(j2\pi k f T_s)$
  - None of the above.
- Let  $m(t)$  be the message signal and  $W$  be the bandwidth of  $m(t)$ . The double-sideband suppressed carrier (DSB-SC) amplitude modulation has the modulated signal  $u(t) = A_c m(t) \cos(2\pi f_c t)$ . Let the received signal be  $r(t) = u(t)$ . Suppose we demodulate the received signal by first multiplying  $r(t)$  by a locally generated signal  $\cos(2\pi f_c t + \phi)$ , and then passing the product signal through an ideal low pass filter with bandwidth  $W$ . What will happen when  $\phi = \pi/2$ ?
  - The demodulated signal is zero.
  - The demodulated signal has the maximum signal strength.
  - The demodulated signal has the best signal-to-noise ratio.
  - The demodulated signal has large frequency components.
  - None of the above.
- For both FM and PM with sinusoidal message signal, we have  $u(t) = A_c \cos(2\pi f_c t + \beta \sin(2\pi f_m t))$ . Let  $J_n(\beta)$  be the Bessel function of the first kind of order  $n$ . What is the Fourier transform of  $u(t)$ ?
  - $\frac{A_c}{2} \sum_{n=-\infty}^{\infty} J_n(\beta) (\delta(f - n f_m) + \delta(f + n f_m))$
  - $\sum_{n=-\infty}^{\infty} A_c J_n(\beta) \cos(2\pi(f_c + n f_m))$
  - $\frac{A_c}{2} \sum_{n=-\infty}^{\infty} J_n(\beta) (\delta(f - f_c - n f_m) + \delta(f + f_c + n f_m))$
  - $\sum_{n=-\infty}^{\infty} A_c J_n(\beta) \sin(2\pi(f_c + n f_m))$
  - None of the above.

5. Determine the power spectral density of  $m(t) = a \cos(2\pi f_m t)$ .

- (a)  $\frac{a^2}{4}\delta(f - f_m) + \frac{a^2}{4}\delta(f + f_m)$
- (b)  $\frac{a^2}{2}\delta(f - f_m) + \frac{a^2}{2}\delta(f + f_m)$
- (c)  $\frac{a}{2}\delta(f - f_m) + \frac{a}{2}\delta(f + f_m)$
- (d)  $a^2\delta(f - f_m) + a^2\delta(f + f_m)$
- (e) None of the above.

6. A digital communication system generates the transmitted signal  $u(t) = \sum_{n=-\infty}^{\infty} a_n h(t - nT_b)$ , where  $a_n$  is an i.i.d. random process with  $P\{a_n = +1\} = P\{a_n = -1\} = 1/2$ , and  $T_b$  is the symbol period. Define  $H(f)$  as the Fourier transform of  $h(t)$ . Determine the power spectral density of  $u(t)$ .

- (a)  $\frac{1}{T_b}H(f)$
- (b)  $\frac{1}{T_b}|H(f)|^2$
- (c)  $T_b|H(f)|^2$
- (d)  $T_bH(f)$
- (e) None of the above.

7. Let  $X(f)$  be the Fourier transform of  $x(t)$ . Which of the following is a necessary and sufficient condition for  $x(t)$  to satisfy

$$x(nT) = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$

- (a)  $\sum_{n=0}^{\infty} X\left(f + \frac{n}{T}\right) = T$
- (b)  $\sum_{n=-\infty}^{\infty} X(f + nT) = T$
- (c)  $\sum_{n=0}^{\infty} X(f + nT) = T$
- (d)  $\sum_{n=-\infty}^{\infty} X\left(f + \frac{n}{T}\right) = T$
- (e) None of the above.

8. A binary communication system has the received signal

$$r(t) = s(t) + n(t) \quad \text{for } 0 \leq t \leq T$$

where  $s(t)$  is the transmitted signal with  $P\{s(t) = s_A(t)\} = P\{s(t) = s_B(t)\} = 1/2$ ,  $n(t)$  is the AWGN with power spectral density  $N_0/2$ , and  $T$  is the symbol period. Assume that the ML receiver is employed. Determine the bit error rate.

- (a)  $Q\left(\sqrt{\frac{d}{2N_0}}\right)$ , where  $d = \int_0^T |s_A(t) - s_B(t)|^2 dt$
- (b)  $Q\left(\sqrt{\frac{d}{2N_0}}\right)$ , where  $d = \int_0^T |s_A(t) - s_B(t)| dt$
- (c)  $Q\left(\sqrt{\frac{d}{N_0}}\right)$ , where  $d = \int_0^T |s_A(t) - s_B(t)|^2 dt$
- (d)  $Q\left(\sqrt{\frac{d^2}{2N_0}}\right)$ , where  $d = \int_0^T |s_A(t) - s_B(t)|^2 dt$
- (e) None of the above.

9. Two signal vectors are employed for a binary communication system which are given by

$$\mathbf{s}_1 = (+A, +A)$$

$$\mathbf{s}_2 = (+A, -A)$$

The received signal can be represented as

$$\mathbf{r} = \mathbf{s} + \mathbf{w}$$

where  $P\{\mathbf{s} = \mathbf{s}_1\} = P\{\mathbf{s} = \mathbf{s}_2\} = 1/2$  and  $\mathbf{w} = [w_1, w_2]$  is a 2-dimensional zero-mean Gaussian noise vector with  $E[|w_i|^2] = N_0/2$  for  $i = 1, 2$  and  $E[w_1 w_2] = 0$ . What is the bit error rate for the optimum detector?

- (a)  $Q\left(\frac{A^2}{N_0}\right)$
  - (b)  $Q\left(\sqrt{\frac{2A^2}{N_0}}\right)$
  - (c)  $Q\left(\sqrt{\frac{A^2}{2N_0}}\right)$
  - (d)  $Q\left(\sqrt{\frac{A^2}{N_0}}\right)$
  - (e) None of the above.
10. Under the same signal-to-noise ratio, which of the following binary modulation techniques has the best performance.
- (a) Binary pulse position modulation.
  - (b) Binary frequency-shift keying.
  - (c) On-off keying.
  - (d) Binary phase-shift keying.
  - (e) None of the above.

**II. Long Questions**

Give detailed derivations on the following questions. The grade of this portion depends not only on the correct answers but also on the explanations and derivations. Therefore, explain every detail as possible as you can.

1. (25%) Consider the case of binary PAM signal over the AWGN channel with received signal

$$r = s + w.$$

where  $s$  is the transmitted signal and  $w$  is the AWGN with variance  $N_0/2$ . The two possible signal points are  $s_1 = -s_2 = \sqrt{E_b}$ , where  $E_b$  is the energy per bit. The prior probabilities are  $P[s = s_1] = p$  and  $P[s = s_2] = 1 - p$ .

- (a) (10 %) For the optimum detector, a threshold  $\lambda$  can be used to make decision on the transmitted signal. If  $r > \lambda$ , the decision  $s = s_1$  is made and if  $r < \lambda$ , the decision  $s = s_2$  is made. Determine the threshold  $\lambda$ .
- (b) (15 %) Given the optimum detector, what is the bit error probability?
2. (25%) The discrete sequence

$$r_k = \sqrt{E_c} c_k + n_k, \quad \text{for } k = 1, 2, \dots, n$$

represents the output sequence of samples from a demodulator, where  $c_k = \pm 1$  are elements of one of two possible codewords  $\mathbf{c}_1 = [1, 1, \dots, 1]$  and  $\mathbf{c}_2 = [1, 1, \dots, 1, -1, -1, \dots, -1]$ . The codeword  $\mathbf{c}_1$  is an all-one vector and  $\mathbf{c}_2$  has  $w$  elements which are +1 and  $n - w$  elements which are -1, where  $w$  is some positive integer. The noise sequence  $\{n_k\}$  is white Gaussian with variance  $\sigma^2$ .

- (a) (10%) What is the maximum-likelihood detector for the two possible transmitted signals?
- (b) (10%) Determine the error probability as a function of the parameter  $(\sigma^2, E_b, w)$ .
- (c) (5%) What is the value of  $w$  that minimizes the error probability?