考試科目	微積分	所別	應用數學系	考試時間	3 月 14 日 星期六 第 1 節

1. (15%) Let $f\colon [0,5] \to \mathbb{R}$ be a function defined by

$$f(x) = \begin{cases} 1 & \text{if } x \neq 1 \\ 0 & \text{if } x = 1. \end{cases}$$

Prove that $f(x)$ is Riemann integrable on $[0,5]$ and evaluate its integral on $[0,5]$.

2. (15%) Let $\sum_{n=1}^{\infty} a_n$ be a series with nonnegative terms and s_n be its n-th partial sum. Show that $\sum_{n=1}^{\infty} a_n$ converges if and only if $\{s_n\}$ is a bounded sequence.

3. (15%) Let $f\colon [a,b] \to \mathbb{R}$ be a continuous function. Show that there exists $c \in [a,b]$ such that

$$\int_a^b f(t)\,dt = f(c)(b-a).$$

4. (15%) Suppose that $f(x)$ is a continuous real-valued function on the real line $\mathbb{R}$. Show that if $f(x+y) = f(x) + f(y)$ for all $x, y \in \mathbb{R}$, then there exists a constant a such that $f(x) = ax$ for all $x \in \mathbb{R}$.

5. (20%) Prove the following identities.

(a) $\displaystyle\int_x^1 \frac{dt}{1+t^2} = \int_1^{1/x} \frac{dt}{1+t^2}$ for $x > 0$.

(b) $\displaystyle\int_0^1 x^m(1-x)^n\,dx = \int_0^1 x^n(1-x)^m\,dx.$

6. (20%) State the Green's Theorem and show that the area of the region $R = \{(x,y) \in \mathbb{R}^2 \mid x^2+y^2 \leq 1\}$ can be expressed as a line integral over the circle $x^2+y^2 = 1$ in the counterclockwise direction.

備考	試題隨卷繳交

命題委員：　　　　　　　　　　　（簽章）

命題紙使用說明：1.試題將用原件印製，敬請使用黑色墨水正楷書寫或打字（紅色不能製版請勿使用）。
2.書寫時請勿超出格外，以免印製不清。
3.試題由郵寄遞者請以掛號寄出，以免遺失而示慎重。