考試科目	線性代數	所別	應用數學系	考試時間	3月14日 星期六 第2節

1. (20%) Let A, B be two $n \times n$ complex matrices such that $AB = BA$. Suppose A has n distinct eigenvalues. Show that B is diagonalizable.

2. (20%) Let A be a 5×5 real matrix. Suppose that $A^3 = 0$, but $A^2 \neq 0$. Find all possible Jordan canonical form for A.

3. (20%) Let W_1 and W_2 be subspaces of a finite-dimensional inner product space V over a field $\mathbb{F}$. Suppose $\dim W_1 = \dim W_2$. Show that there is an orthogonal linear operator T on V such that $T(W_1) = T(W_2)$.

4. (20%) Let V and W be finite-dimensional vector spaces over a field $\mathbb{F}$ with ordered bases $\beta = \{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n\}$ and $\gamma = \{\mathbf{w}_1, \mathbf{w}_2, \ldots, \mathbf{w}_m\}$, respectively. Let $\mathcal{L}$ be the vector space of all linear transformation from V to W. Suppose that $T_{ij} : V \to W$ is the linear transformation such that

$$T_{ij}(\mathbf{v}_k) = \begin{cases} \mathbf{w}_i & \text{if } k = j \\ 0 & \text{if } k \neq j. \end{cases}$$

Show that $S = \{T_{ij} \mid 1 \leq i \leq m, 1 \leq j \leq n\}$ is a basis for $\mathcal{L}$.

5. (20%) Let V and W be two subspaces of a vector space over a field $\mathbb{F}$. Show that $V/(V \cap W)$ is isomorphic to $(V + W)/W$. Please prove directly by the definition of two vector spaces being isomorphic.

備考	試題隨卷繳交
命題委員：	（簽章）