※選擇題請在答案卡內作答，非選擇題請在答案卷內作答

一、單選題 (共 10 題，每題 4 分，每題有五個選項。答錯倒扣 1 分)

1. A passenger car is running at 50 km/h. A sports car is overtaking this passenger car now. If this sports car is 50 km/h as seen from the person riding in the passenger car, the speed of this sports car as seen from the person standing on the sidewalk is 100 km/h. Here is a problem; a passenger car is running at two-thirds speed of light speed (200,000 km/s). A sports car is overtaking this passenger car now. If the sports car is 200,000 km/s as seen from the person riding in the passenger car, what is the speed of this sports car as seen from the person standing on the sidewalk? The speed of light is about 300,000 km/s.
(A) About 200,000 km/s
(B) About 400,000 km/s
(C) About 280,000 km/s
(D) About 300,000 km/s
(E) About 250,000 km/s

2. Choose the correct combination of the answers in the parentheses below.
In radiation from a blackbody, the ( a ) at which the spectral radiation becomes maximum is related inversely to the absolute temperature of the blackbody. This rule is called ( b ). For example, suppose that an object is heated and emits light. When you compare the cases that the object looks red and blue, the temperature is higher when it looks ( c ). In the case of an incandescent lamp with a color temperature of 2900 K, the wavelength at which the spectral radiation is maximum is approximately ( d ) nm.
(A) (a) frequency, (b) Stefan–Boltzmann law, (c) red, (d) 500
(B) (a) frequency, (b) Wien's displacement law, (c) red, (d) 500
(C) (a) wavelength, (b) Stefan–Boltzmann law, (c) blue, (d) 1000
(D) (a) wavelength, (b) Wien's displacement law, (c) blue, (d) 1000
(E) (a) wavelength, (b) Stefan–Boltzmann law, (c) blue, (d) 500

3. If an electron is accelerated through V volts then the de Broglie wavelength in angstroms is given by $\lambda =$?
(A) $(15/V)^{0.5}$
(B) $(1500/V)^{0.5}$
(C) $(150/V)^{0.5}$
(D) $(1.5/V)^{0.5}$
(E) $(15000/V)^{0.5}$

4. Ultraviolet light of wavelength 350 nm and intensity 0.1 $W/m^2$ is directed at a potassium surface. If only 0.5% of the incident photons produce photoelectrons, how many are emitted per second if the potassium surface has an area of 1 $cm^2$? The work function of potassium is about 2.2 eV.
(A) $8.8\times10^{10}$
(B) $8.8\times10^{11}$
(C) $3.5\times10^{10}$
(D) $3.5\times10^{11}$
(E) $1.3\times10^{11}$

注意：背面有試題

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5. Assume that $m_e$ is the electron mass, $n$ is the principal quantum number, $c$ is the speed of light, and $\alpha$ is the fine structure constant. The energy $E_n$ of positronium is given by
(A) $-\alpha^2 m_e c^2/4n^2$
(B) $-\alpha^2 m_e c^2/2n^2$
(C) $-\alpha^2 m_e c^2/n^2$
(D) $-2\alpha^2 m_e c^2/n^2$
(E) $-4\alpha^2 m_e c^2/n^2$

6. Select the correct solution of wave function, eigen energy, and/or eigen value for angular momentum, associated with various conditions of quantum well and/or potential barriers
(A) A particle in a quantum well with width of $L$ and barrier height of $U_0$ at the boundaries, $x = 0, L$
$$\psi_n(x) = \sqrt{\frac{2\pi}{L}}\sin\left(\frac{n\pi}{L}x\right), n = 1,2,3,\cdots, E_n = \frac{n^2(\pi\hbar)^2}{2mL^2}, n = 1,2,3,\cdots$$
(B) For harmonic oscillator with potential energy of $U$ and quantized energies of $E_n$
$U(x) = \frac{1}{2}kx^2$, $E_n = n\hbar\omega, n = 0,1,2,3,\cdots$
wave function: $\psi(y) = (2^n n!)^{-1/2} H_n(y) e^{\frac{-y^2}{2}}, y = (\frac{m\omega}{\hbar})^{1/2}x$, $\omega = \sqrt{\frac{k}{m}}$
$H_n(y)$: Hermite Polynomials
(C) For Hydrogen atom with mass of $M_H$, potential energy of $U(r)$ and quantized energies of $E_n$
$U(r) = -\frac{e^2}{4\pi\varepsilon_0 r}$ $E_n = \frac{m_e e^4}{8\varepsilon_0^2 h^2}\left(\frac{1}{n^2}\right), n = 1,2,3,\cdots$
angular momentum: $L = mvr = n\hbar$, $\hbar = \frac{h}{2\pi}$, $h$: Planck constant
(D) For an electron with energy of $E$, tunneling through a barrier with barrier height of $U_0$ and thickness of $\delta$
wave function: $\psi(x) = e^{-k\delta}$, tunneling probability: $T = \int_{-\infty}^{\infty}\psi^*\psi dx = e^{-2k\delta}$, $k = \frac{\sqrt{2m(U_0-E)}}{\hbar}$
(E) None of the above

7. Consider an atomic electron in the $l = 3$ state. ( $l$ is the orbital quantum number.) Which of the following is the magnitude of orbital angular momentum $L$ in unit of Planck constant $\hbar$?
(A) 3
(B) (3/2)
(C) $2\sqrt{3}$
(D) $\sqrt{3}$
(E) 1/2

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8. According to Drude's free electron theory of metals, the physical properties of a metal may be explained by modeling the metal as a classical gas of conduction electrons moving through a fixed lattice of positive ion cores. Which of the following is NOT true for the original Drude's theory?
(A) The model predicts correct functional form of Ohm's law
(B) The drift velocity of the electron is proportional to the applied electrical field
(C) The original model uses classical mean free path
(D) The original model predicts correctly the experimental values of electrical conductivities with classical mean free path
(E) The mobile electrons are visualized as "fluid"

9. For Hydrogen atom, select the correct wave function at given quantum numbers to accurately calculate the expectation values.
(A) For wave function at ground state $\Psi_{100}(r,\theta,\phi)$ to calculate the $\langle r\rangle$ away from the nucleus
$\Psi_{100}(r,\theta,\phi) = R_{100}(r)\Theta_{100}(\theta)\Phi_{100}(\phi) = \frac{1}{a_0{}^{3/2}} e^{-r/a_0}$, $a_0 = \frac{4\pi\varepsilon_0\hbar^2}{me^2}$ (Bohr radius)
$\langle r\rangle = a_0, \frac{1}{\langle r\rangle} = \frac{1}{a_0}$

(B) For wave function at ground state $\Psi_{100}(r,\theta,\phi)$ to calculate the $\langle\frac{1}{r}\rangle$
$\Psi_{100}(r,\theta,\phi) = R_{100}(r)\Theta_{100}(\theta)\Phi_{100}(\phi) = \frac{1}{\sqrt{\pi}a_0{}^{3/2}} e^{-r/a_0}$, $a_0 = \frac{4\pi\varepsilon_0\hbar^2}{me^2}$ (Bohr radius)
$\langle\frac{1}{r}\rangle = \frac{1}{\pi}\frac{1}{a_0}$

(C) For wave function at ground state $\Psi_{100}(r,\theta,\phi)$ to calculate the $\langle r\rangle$ and $\langle\frac{1}{r}\rangle$
$\Psi_{100}(r,\theta,\phi) = R_{100}(r)\Theta_{100}(\theta)\Phi_{100}(\phi) = \frac{1}{\sqrt{\pi}a_0{}^{3/2}} e^{-r/a_0}$, $a_0 = \frac{4\pi\varepsilon_0\hbar^2}{me^2}$ (Bohr radius)
$\langle r\rangle = a_0, \langle\frac{1}{r}\rangle = \frac{1}{a_0} = \frac{1}{\langle r\rangle}$

(D) For wave function at excited state $\Psi_{200}(r,\theta,\phi)$ to calculate the $\langle\frac{1}{r}\rangle$
$\Psi_{200}(r,\theta,\phi) = R_{200}(r)\Theta_{200}(\theta)\Phi_{200}(\phi) = \frac{1}{4\sqrt{2\pi}a_0{}^{3/2}}(2-\frac{r}{a_0})e^{-r/2a_0}$, $a_0 = \frac{4\pi\varepsilon_0\hbar^2}{me^2}$
$\langle\frac{1}{r}\rangle = \frac{1}{4a_0}$

(E) For wave function at excited state $\Psi_{200}(r,\theta,\phi)$ to calculate the $\langle r\rangle$ and $\langle\frac{1}{r}\rangle$
$\Psi_{200}(r,\theta,\phi) = R_{200}(r)\Theta_{200}(\theta)\Phi_{200}(\phi) = \frac{1}{2\sqrt{2\pi}a_0{}^{3/2}}(2-\frac{r}{a_0})e^{-r/2a_0}$, $a_0 = \frac{4\pi\varepsilon_0\hbar^2}{me^2}$
$\langle r\rangle = 2a_0, \langle\frac{1}{r}\rangle = \frac{1}{2a_0} = \frac{1}{\langle r\rangle}$

注意：背面有試題

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10. X-rays of wavelength 10 pm are scattered from a target. Find the maximum wavelength present in the scattered X-rays. Compton wavelength of the scattering particle is about 2.426 pm.
(A) About 12 pm
(B) About 15 pm
(C) About 20 pm
(D) About 30 pm
(E) About 40 pm

二、複選題(共 10 題，每題 4 分，每題有五個選項。答錯倒扣 0.8 分)

11. Choose the correct statements below.
(A) No phenomenon exists which appears to be a speed (super light speed) exceeding the speed of light in vacuum.
(B) The principle of relativity is that physical laws are represented by the same formula even for different observers.
(C) In Minkowski's spacetime, the quantity $(ct)^2 + x^2 + y^2 + z^2$ is invariant under the Lorentz transformation. Here the coordinates $x$, $y$, $z$ refer to space, $t$ is time, and $c$ is the speed of light.
(D) Massless objects can only move at the speed of light.
(E) Special relativity theory can only deal with uniform linear motion.

12. Choose the correct statements below. $c$ is the speed of light.
(A) A spaceship is measured to be 100 m long while it is at rest with respect to an observer. If this spaceship flies by the observer with a speed of 0.99$c$, the observer will find the length of the spaceship to be about 14.1 m.
(B) There is a twin, **A** and **B** on the earth. Suppose when they are 20 years old, **A** makes a round trip to the Centaurus α star that is 4.4 light-years far away from the earth by spacecraft at the speed of $v = 0.99c$. When **A** returns the earth and see **B**, **A** and **B** will be about 21.25 and 28.9 years old, respectively.
(C) An electron, which has a mass of $9.11\times10^{-31}$ kg, moves with a speed of 0.750$c$. Its relativistic momentum is $2.05\times10^{-22}$ kg·m/s.
(D) X-rays of wavelength $\lambda = 0.2$ nm are aimed at a block of carbon. The scattered X-rays are observed at an angle of 45° to the incident beam. The wavelength of the scattered X-ray at this angle is 0.1993 nm.
(E) An electron of charge q and mass m is accelerated from rest through a small potential difference V = 50 V. Assuming that the particle is nonrelativistic, de Broglie wavelength λ is 1.7 Å.

注意：背面有試題

※選擇題請在答案卡內作答，非選擇題請在答案卷內作答

13. Which of the following statements are correct?
    (A) A wire of length 1m and radius 1mm is heated via an electric current to produce 1 kW of radiant power. Treating the wire as a perfect blackbody and ignoring any end effects, the temperature of the wire is 1294 K.
    (B) If $u$ is the energy density of radiation, the radiation pressure is given by $u/3$.
    (C) In the quantum theory of blackbody radiation Planck assumed that the oscillators are allowed to have energy, 0, ε, 2ε, ⋯ The mean energy of the oscillator is given by $\varepsilon/(e^{\varepsilon/kT}-1)$, where $\varepsilon = h\nu$.
    (D) The kinetic energy (in eV) of an electron in a hydrogen atom calculated by using the uncertainty principle is in agreement with the result obtained from Bohr's theory of hydrogen atom.
    (E) The photoelectric effect cannot take place with a free electron.

14. The state of a free particle is described by the wave function: $\Psi(x) = 0$ for $x < -3a$, $\Psi(x) = c$ for $-3a < x < a$, and $\Psi(x) = 0$ for $x > a$. Which of the following statements are correct?
    (A) If we are using the normalization condition, $c = 1/2a^{1/2}$.
    (B) The probability of finding the particle in the interval $[0, a]$ is 1/2.
    (C) The expectation value of $x$ is $-a$ and the standard deviation of $x$ is $2a/3^{1/2}$.
    (D) The expectation value of $x$ is $7a^2/3$ and the variance of $x$ is $2a/3^{1/2}$.
    (E) If the momentum probability density is $|\varphi(p)|^2$, $|\varphi(p)|^2 = \frac{\hbar}{2\pi a p^2}\sin^2(\frac{2}{\hbar}pa)$

15. Consider an atomic electron in the $l = 3$ state. ($l$ is the orbital quantum number.) Which of the following are allowable values of $z$ component of orbital angular momentum $L$ in unit of Planck's constant $\hbar$.
    (A) 0
    (B) 1
    (C) -2
    (D) 3
    (E) 3/2

16. For free electron gas in metals, which of the following is true?
    (A) The Fermi temperature $T_f$ is defined by $E_f = k_B T_f$, where $E_f$ is Fermi energy and $k_B$ is the Boltzmann constant.
    (B) For typical metal like aluminum in ambient condition, Fermi energy is on the order of 0.03 eV
    (C) For typical metal like aluminum in ambient condition, Fermi temperature is ~300K.
    (D) Pauli exclusion principle can be applied to such a system.
    (E) For typical metal like sodium, Fermi temperature is much larger than 1000K.

注意：背面有試題

※選擇題請在答案卡內作答，非選擇題請在答案卷內作答

17. For an electron trapped in a quantum well with infinitely high barrier $U(x) \to \infty$ at the boundaries, $x = 0$ and $L$. Derive the wave function and calculate the expectation values of specified terms. Herein $p$ is the linear momentum operator given by $p = \frac{\hbar}{i}\frac{d}{dx}$

(A) $\Psi(x) = \sqrt{\frac{2\pi}{L}}\cos\frac{n\pi x}{L}$, $\langle x \rangle = \frac{L}{2}$, $\langle x^3 \rangle = \frac{L^3}{8}$, $\langle x^3 \rangle - \langle x \rangle^3 = 0$

(B) $\Psi(x) = \sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L}$, $\langle x \rangle = \frac{L}{2}$, $\langle x^3 \rangle = \frac{L^3}{4}$, $\langle x^3 \rangle - \langle x \rangle^3 = \frac{L^3}{8}$

(C) $\Psi(x) = \sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L}$, $\langle px^2 \rangle - \langle x^2 p \rangle = \frac{\hbar}{i}L$

(D) $\Psi(x) = \sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L}$, $\langle px \rangle - \langle xp \rangle = 0$

(E) $\Psi(x) = \sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L}$, $\langle px^3 \rangle - \langle x^3 p \rangle = \frac{\hbar}{i}L^2\left(1 - \frac{3}{2(n\pi)^2}\right)$

18. An electron magnetic dipole was placed in a magnetic field B. Calculate the minimum $B$, denoted as $B_0$ needed for the Zeeman effect to be observed in a spectral line of various wavelengths, such as λ = 490 nm, 500 nm, and 600 nm corresponding to spectrometer with resolution of Δλ = 0.084 nm, 0.056 nm, and 0.045 nm, respectively.

(A) For λ = 490 nm, Δλ = 0.084 nm, $B_0$ = 6.0 T (T = kg/$C$·$s$)
(B) For λ = 490 nm, Δλ = 0.084 nm, $B_0$ = 7.5 T (T = kg/$C$·$s$)
(C) For λ = 500 nm, Δλ = 0.056 nm, $B_0$ = 5.0 T (T = kg/$C$·$s$)
(D) For λ = 600 nm, Δλ = 0.045 nm, $B_0$ = 2.68 T (T = kg/$C$·$s$)
(E) For λ = 600 nm, Δλ = 0.045 nm, $B_0$ = 4.5 T (T = kg/$C$·$s$)

19. Which of the following statements are correct?

(A) All motion is relative; the speed of light in free space is the same for all observers.
(B) A spacecraft is moving relative to the earth. An observer on the earth finds that, between 1 P.M. and 2 P.M. according to her clock, 3601 s elapse on the spacecraft's clock. The spacecraft's speed relative to the earth is around $7\times10^6$ m/s.
(C) To accelerate a particle of mass m and velocity $v$ with a constant force F, where F is parallel to $v$, the acceleration of the particle is $a = \frac{F}{\gamma m}$. ($\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$)
(D) A particle has the rest mass $m_0$ and velocity v. The speed of light is c. Therefore, the relativistic momentum is $p = \frac{m_0 v}{\sqrt{1-\frac{v^2}{c^2}}}$ and relativistic total energy is $E = \frac{m_0 c^2}{\sqrt{1-\frac{v^2}{c^2}}}$
(E) The energies of electrons liberated by light depend on the frequency of the light.

Electron mass $m_e$=9.1095× $10^{-31}$ kg; Hydrogen atomic mass $M_H$=1.6736× $10^{-27}$ kg
Electron charge $e$=1.602× $10^{-19}$ Coul; Planck's constant $h$=6.626× $10^{-34}$ J·s
Light velocity $c$=3.00× $10^8$ m/s; Permittivity of free space $\varepsilon_0$=8.854× $10^{-12}$ Coul/V·m

注意：背面有試題

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20. Which of the following statements are correct?
(A) Pair production can occur in free space.
(B) In an X-ray tube, the higher the accelerating voltage V, the faster the electrons and the shorter the wavelengths of the X-rays can be obtained.
(C) The maximum photoelectron energy $KE_{max}$ is proportional to the frequency $v$ of the incident light
(D) In blackbody spectra, the spectral distribution of energy in the radiation depends only on the temperature of the body.
(E) In Compton scattering, the greater the scattering angle, the greater the wavelength change.

三、非選題 (共 20 分)

21. (10%) Metal free electron model. At T = 0K, a metal has $N$ electrons in volume $V$.
(A) (5%) Derive the density of state function $g(E) = \frac{8\sqrt{2}\pi V m^{3/2}}{h^3}\sqrt{E}$
(B) (5%) Determine the Fermi energy $E_F$.

22. (10%) The solid state crystal
(A) (5%) Explain the following figure of Carbon and Silicon energy band versus internuclear distance.
(B) (5%) When T = 0K, T = 300K, what is the difference of electrons distribution of conduction band and valence band for Carbon and Silicon, respectively.