考 試 科 目	統計學	所 別	經濟學系 261	考試時間	3 月 6 日(六) 第 四 節

1. A random sample of size n, $Y_1, Y_2, \ldots, Y_n$, is taken from the pdf

$$f_Y(y;\theta_o) = cy\theta_o^2,\ 0 \le y \le \frac{1}{\theta_o},$$

where c is a constant and θ_o is the unknown parameter of interest. Let $\hat{\theta}_{mm}$ denote the Method of Moments estimator for θ_o, and $\hat{\theta}_{ml}$ the Maximum Likelihood estimator for θ_o.

   (a) Find $\hat{\theta}_{mm}$ and $\hat{\theta}_{ml}$. (5%)

   (b) Is $\hat{\theta}_{ml}$ unbiased? (5%)

   (c) Show the Cramér-Rao lower bound in this case. (5%)

   (d) Is it possible that the variance of an unbiased estimator is less than the derived Cramér-Rao lower bound in (c)? Why or Why not? (5%)

   (e) Is $\hat{\theta}_{mm}$ a sufficient estimator for θ_o? Why or Why not? (5%)

   (f) Is $\hat{\theta}_{ml}$ a consistent estimator for θ_o? Why or Why not? (5%)

2. Let $X_1, X_2, \ldots, X_n$ and $Y_1, Y_2, \ldots, Y_m$ be independent random samples from normal distributions with mean μ_X and μ_Y and standard derivations σ_X and σ_Y, respectively.

   (a) If σ_X and σ_Y are known, derive a $100(1-\alpha)\%$ confidence interval for $\mu_X - \mu_Y$. (5%)

   (b) For testing $H_0; \sigma_X^2 = \sigma_Y^2$ versus $H_1 : \sigma_X^2 \neq \sigma_Y^2$,

      (i) Derive the likelihood ratio test statistic in detail. (5%)

      (ii) Explain how to implement the likelihood ratio test given the significance level α. (5%)

3. Show the "**memoryless property**" of the geometric random variable X. (5%)

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4. In the model $y_t = \alpha + \beta x_t + e_t$, with x_t non-stochastic. Assume that $E(e_i) = 0$ and $E(e_i^2) = \sigma_0^2$. Giving the following sample moments:

$$\sum_{t=1}^{10} y_t = 8,\ \sum_{t=1}^{10} x_t = 40,\ \sum_{t=1}^{10} y_t^2 = 26,\ \sum_{t=1}^{10} x_t^2 = 200,\ \sum_{t=1}^{10} x_t y_t = 20.$$

Assume that this model holds for $x_{11} = 10$ and $x_{12} = 12$.

(a) Calculate the best linear unbiased predictor of y_{11} and y_{12}. (5%)

(b) Estimate the standard error of your forecast in (a). (10%)

(c) If the realized values for y_{11} and y_{12} are 0.5 and 0.6 respectively, test the null hypothesis that $H_0 : E(e_{11}) = 0$ and $H_0 : E(e_{12}) = 0$ at the 5% level. State additional assumption you need to carry the test. (15%)

5. Let $e_0, e_1, \cdots, e_T$ be a sequence of independent and identically distributed $N(0, \sigma_0^2)$ random variables for some σ_0^2. Assume that

$$y_t = \alpha_0 + \beta_0 e_{t-1} + e_t,\ t = 1, \cdots, T,$$

for some α_0 and β_0.

(a) Please derive the variance of y_t. (5%)

(b) Please derive the autocovariances, $cov(y_T, y_{T-k})$, $k = 1, 2, \cdots, T-1$. (5%)

6. Assume that $y_t = \beta_1 x_{t1} + \beta_2 x_{t2} + u_t$, $E(u_t) = 0, E(u_t^2) = \sigma^2$, $E(u_t u_s) = 0$. All variables have zero mean. If β_1 is estimated from the regression of y on x_1 with x_2 omitted, show that the resulting estimate is biased but has smaller variance than the estimate with x_2 included. (10%)

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