

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Solve the non-homogenous Euler-Cauchy Equation. (25%)

$$xy'' - \frac{2}{x}y = 1$$

2. Using the convolution theorem and Laplace transform solve the equation. (25%)

$$y(t) = te^t - 2e^t \int_0^t e^{-x} y(x) dx$$

3. Matrix $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & -1 \end{bmatrix}$

(1) Find the inverse matrix A^{-1} (10%)

(2) Find the eigenvalues and the corresponding eigenvectors of A . (10%)

(3) Find two nonsingular matrices Q and P such that QAP is a diagonal matrix. (5%)

4. For partial differential equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} ; u(0,t) = u(\pi,t) = 0 ; u(x,0) = 10 \sin x$$

Find the solution. (25%)