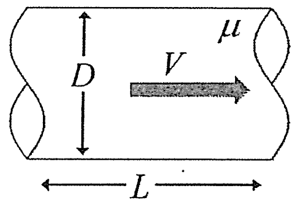
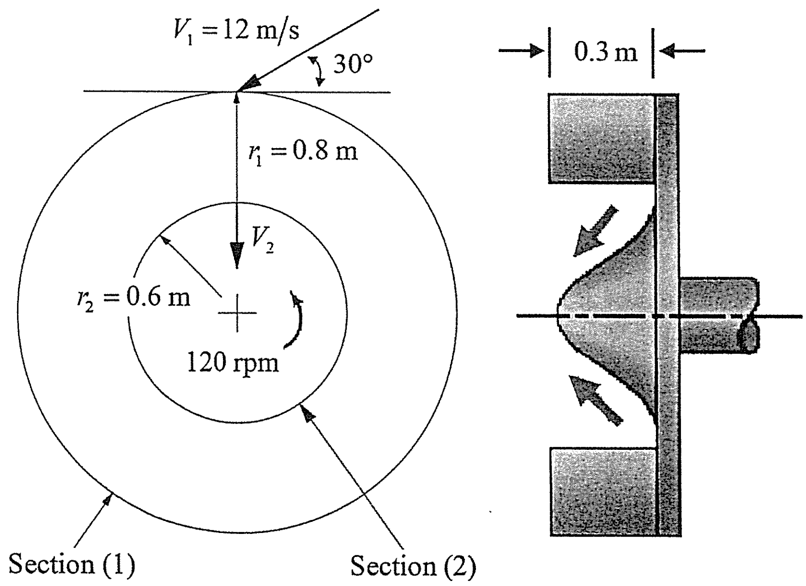
※ 考生請注意：本試題可使用計算機。 請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

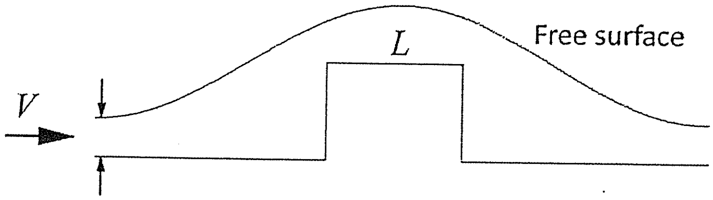
1. Considering a horizontal circular pipe with a steady, incompressible, fully developed laminar flow and a section of length equal to L, where the viscosity of the fluid is μ.
   (a) (10%) Please determine the pressure difference Δp (by the length L, the pipe diameter D, and wall shear stress τ_w).
   (b) (10%) Let V be the average velocity, please show that $V = \dfrac{\Delta p D^2}{32\mu L}$.
   (c) (10%) With (b) please show that the head loss reads
   $h_L = f\dfrac{V^2}{2g}\dfrac{L}{D}$ with $f = \dfrac{64}{\mathrm{Re}}$.



2. For a water turbine with radial flow, the absolute entering velocity is 12 m/s, and it makes an angle of 30 degree with the tangent to the rotor. The absolute exit velocity is directed radially inward. The angular speed of the rotor is 120 rpm. (ρ_w = 1,000 kg/m^3.)
   (a) (10%) Please find the radial component of velocity at section (1) and (2).
   (b) (10%) Please find the power delivered to the shaft of the turbine.



3. A thin layer of an incompressible fluid flows steadily on a horizontal smooth plate and over an obstruction with a square cross section, where ρ is the flow density, μ the viscosity and σ is the surface tension. A model with a length scale of 1/5 and a fluid density scale of 1.0 is to be designed to predict the depth of fluid, y, along the plate. Assume that the inertial, gravitational, surface tension and viscous effects are all important. With the help of Froude number $\mathrm{Fr} = V/\sqrt{gL}$, Reynolds number $\mathrm{Re} = \rho VL/\mu$ and Weber number $\mathrm{We} = \rho V^2 L/\sigma$, please determined the required viscosity scale μ_m/μ and surface tension scale σ_m/σ? (20%)



4. For an incompressible flow (density ρ) through a straight pipe with radius R. At section (1) the distribution of velocity is uniform $u_1 = U$, and the velocity profile is symmetric and parabolic, i.e. $u_2 = u_M\left(1 - r^2/R^2\right)$ at section (2). The distance between the two sections is L.
   (a) (10%) How is U and u_M related? And how is U and the averaged velocity $\overline{u}_2$ at section (2) related?
   (b) (10%) Letting the wall friction (per unit area) be given by f_r, please express the pressure drop between sections (1) and (2), $\Delta p = p_1 - p_2$.
   (c) (10%) Please prove that the energy loss (per unit mass flow rate) from section (1) to section (2) is $\dfrac{2 f_r L}{\rho R} - \dfrac{\overline{u}_2^{\,2}}{6}$.