※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Consider the equation $y'' + 5y' + 6y = u(t-1) - u(t-2)$, $y(0) = 1$, and $y'(0) = 0$,

   where $u(t-a) = \begin{cases} 0 & 0 \le t < a, \\ 1 & a \le t. \end{cases}$　　Solve for $y(t)$ with Laplace transform.　　(20 points)

2. (a) Find the tangent to the ellipse $\dfrac{1}{4}x^2 + y^2 = 1$ at $P:(\sqrt{2},\ 1/\sqrt{2})$　　(6 points)

   (b) Using Green's theorem, evaluate the line integral $\oint_C \mathbf{F(r)} \bullet d\mathbf{r}$ counterclockwise around the

   boundary C of the region R

   where $\mathbf{F} = \left[x^2 e^y,\ y^2 e^x \right]$, C the rectangle with vertices $(0, 0), (2, 0), (2, 3), (0, 3)$　　(6 points)

   (c) $f = \dfrac{1}{r}$, $r = \sqrt{x^2 + y^2 + z^2}$, find $\displaystyle\iint_S \nabla f \cdot \hat{n}\, dA$ for S: $x^2 + y^2 + z^2 = a^2$　　(8 points)

（背面尚有題目）

3. (a) Find the singular points and the corresponding residues:　　(10 points)

　　（ⅰ）$\dfrac{1}{z(z-1)^2}$　　（ⅱ）$\cot(z)$

(b) Using the residue theorem, evaluate $I = \displaystyle\int_{-\infty}^{\infty} \dfrac{x+2}{x^3+4x}\,dx$　　(10 points)

4. (a) Define the even and odd functions, respectively.　　(6 points)

(b) Is a Fourier cosine series an even or odd function?

　　How about a Fourier sine series? Show your arguments.　　(4 points)

(c) Show that any function, $f(x)$, can be uniquely decomposed into the sum of an even function and an odd function.　　(10 points)

5. Consider a two-dimensional Laplace's equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \quad \text{in the domain of } 0 \le x \le a \text{ and } 0 \le y \le b$$

with the boundary condition of $\begin{cases} u = y & \text{at} \quad x = 0, \quad 0 \le y \le b \\ u = c & \text{at} \quad x = a, \quad 0 \le y \le b \\ u = c & \text{at} \quad y = 0, \quad 0 \le x \le a \\ \dfrac{\partial u}{\partial y} = 0 & \text{at} \quad y = b, \quad 0 \le x \le a \end{cases}$

Find the solution of $u(x, y)$　　(20 points)