

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (10 %) $x^2\ddot{y} - x\dot{y} + y = 0$, $y(1) = 1, \dot{y}(1) = 2$. Find $y(x)$

(Hint: Euler-Cauchy equations)

2. (10 %) Solve by Laplace Transform,

$$y(t) - \int_0^t y(\tau)\sin 2(t - \tau)d\tau = \sin 2t$$

3. (10 %) Find the Laplace Transform of following functions

$$te^{-t}\cos t + t^2\sin t$$

4. (30 %) The differential equation, $\ddot{y} + 4y = f(t)$ and $y(0) = \dot{y}(0) = 0$

(a) Find the transfer function. Hint $H(s) = \frac{Y(s)}{F(s)}$ (10%)

(b) When $f(t) = u(t) - u(t - 1)$, To solve the $y(t)$ (10%)

Hint: $u(t)$ is a unit step function

(c) When $f(t) = \sin t$, To solve the $y(t)$ (10%)

5. (20 %) When $f(t) = u(t) - u(t - 1)$

(a) Find the Laplace transform of the function $f(t)$ (5 %)

(b) Find the Fourier transform of the function $f(t)$ (5 %)

(c) Using half-range expansion (odd or even periodic extension)

then find it Fourier series. (10 %)

$$\text{Hint: } F(s) = \int_0^{\infty} f(t)e^{-st}dt \quad F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-i\omega t}dt$$

6. (20 %) Find the following plate's moment of inertia with vertical axis at center of gravity.

(a) A homogeneous round plate with radius R , thickness h and total mass M .(10%)

(b) A homogeneous square plate with side length $2R$, thickness h and total mass M .(10%)

(Hint; $I = \int r^2 dm$)