編號： 151　　國立成功大學 108 學年度碩士班招生考試試題

系　所：測量及空間資訊學系

考試科目：線性代數　　考試日期：0223，節次：2

※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1.There are two linear equation systems: (A)$\begin{cases} x_1 + x_2 = 2 \\ x_1 - x_2 = 0 \end{cases}$, and (B) $\begin{cases} y_1 + y_2 = 2 \\ y_1 + 1.001y_2 = 2.001 \end{cases}$.

They represent two straight lines on a plane with an intersection angle of 90° and circa 0°, respectively. Both can be expressed as (A) $\mathbf{A}\,\mathbf{x} = \mathbf{b}$, and (B) $\mathbf{B}\,\mathbf{y} = \mathbf{d}$ with $\mathbf{A}=\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$, $\mathbf{x}=\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, $\mathbf{b}=\begin{bmatrix} 2 \\ 0 \end{bmatrix}$, $\mathbf{B}=\begin{bmatrix} 1 & 1 \\ 1 & 1.001 \end{bmatrix}$, $\mathbf{y}=\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$, $\mathbf{d}=\begin{bmatrix} 2 \\ 2.001 \end{bmatrix}$. Their solutions are x_1=1, x_2=1, y_1=1, y_2=1.

(1a) Please compute the inverse matrix of $\mathbf{A}$ and $\mathbf{B}$, respectively, i.e. A^{-1}=? and B^{-1} =? (8%)

(1b) Please compute the **matrix norms** $\|\mathbf{A}\|$, $\|\mathbf{B}\|$, $\|\mathrm{A}^{-1}\|$, and $\|\mathrm{B}^{-1}\|$, if we use the **l_1-vector norm**, where the l_1-vector norm gives the column "sum" norm. (4%)

(1c) The **condition number** $\kappa(\mathbf{C})$ of a square matrix $\mathbf{C}$ is defined by $\kappa(\mathbf{C}) = \|\mathrm{C}\|\,\|\mathrm{C}^{-1}\|$. Please compute the condition numbers $\kappa(\mathbf{A})$ and $\kappa(\mathbf{B})$ of both matrices $\mathbf{A}$ and $\mathbf{B}$. (4%)

(1d) Is the matrix $\mathbf{A}$ more well-conditioned or ill-conditioning than the matrix $\mathbf{B}$? Why? (3%)

(1e) To see the meaning of $\kappa(\mathbf{A})$ and $\kappa(\mathbf{B})$, we add a small error term ε, e.g. ε=0.0001, to the 1st row and 2nd column entry in $\mathbf{A}$ and $\mathbf{B}$ and get (A) $\mathbf{A'}\,\mathbf{x} = \mathbf{b}$, and (B) $\mathbf{B'}\,\mathbf{y} = \mathbf{d}$ with $\mathbf{A'}=\begin{bmatrix} 1 & 1.0001 \\ 1 & -1 \end{bmatrix}$ and $\mathbf{B'}=\begin{bmatrix} 1 & 1.0001 \\ 1 & 1.001 \end{bmatrix}$. Please compute the new solutions which are rounded off to the sixth place below the decimal point, i.e. x_1'= ? x_2'= ?

$y_1' = ?$ $y_2' = ?$ (4%)

(1f) Comparing these new solutions with the old ones, please compute $\Delta_A = \max(|x_1' - x_1|, |x_2' - x_2|)$, $\Delta_B = \max(|y_1' - y_1|, |y_2' - y_2|)$ and ξ with $\Delta_A : \Delta_B = 1 : \xi$, where the symbol max(a, b) denotes the maximum of the two numbers a and b, i.e. $\Delta_A = ?$ $\Delta_B = ?$ $\xi = ?$ (6%)

Then, you see the former is much less than the latter. It means **a well-conditioned matrix gives a more *stable* solution.**

2. The following linear system:

$$\begin{bmatrix} 1 & 0 & 0 & 2 & -2 & 1 \\ -1 & 2 & 0 & 1 & -1 & 1 \\ 0 & 1 & 2 & -2 & 3 & -1 \\ 2 & 2 & -1 & 0 & 1 & 2 \\ 1 & -1 & 1 & 0 & 2 & 0 \\ 4 & 2 & 1 & 1 & 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} x_6 \\ x_4 \\ x_5 \\ x_1 \\ x_3 \\ x_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \\ 3 \\ -3 \\ 4 \\ -3 \end{bmatrix} \qquad (1)$$

(2a) Please change the sequence of entries in the unknown vector $\mathbf{x}$ to

$$\mathrm{x}^T = [x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5 \quad x_6]$$

and write the linear system in the same form as (1), which can be denoted in matrix by $\mathbf{A}\,\mathbf{x} = \mathbf{b}$. (6%)

(2b) Find the solution vector by **Gauss elimination**. (18%)

(2c) Compute the determinant of $\mathbf{A}$, namely det $\mathbf{A}$ = ? (8%)

(2d) Find the rank of $\mathbf{A}$, namely rank $\mathbf{A}$ = ? (4%)

3.Please find the straight line L_1 through the point P with the coordinates (x, y)=(1,2) in the xy-plane and perpendicular to the straight line L_2: $3x$-$2y$+2=0. (10%)

4.The vector function $\vec{r}(t) = R\cos\omega t\ \vec{i} + R\sin\omega t\ \vec{j}$ represents a circle of radius R with center at the origin of the xy-plane and describes a motion of a particle in the counterclockwise sense, where t denotes a time variable. Please write its velocity vector and acceleration vector functions. (10%)

5.

(5a) Please write a **parametric representation** of the ellipse $x^2 + \frac{1}{4}y^2 = 1$. (4%)

(5b) Please find the parametric representation of the tangent to the ellipse $x^2 + \frac{1}{4}y^2 = 1$ at the point P with the coordinates $(x, y) = (1/\sqrt{2}, \sqrt{2})$. (11%)