

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Consider $y'' + \left(\frac{A}{x}\right)y' + \left(\frac{B}{x^2}\right)y = 0$, where A and B are both constant, $x > 0$ and $(A - 1)^2 - 4B = 0$,
- (a) Please find its general solution. (10%)
 - (b) Please discuss and evaluate y if $x \rightarrow 1.0$. (5%)
 - (c) Please discuss and evaluate y if $A \rightarrow 1.0$. (5%)

2. Define the discrete convolution sum of functions f and g as (20%)
- $$h(kT) = \sum_{j=0}^{N-1} f(jT)g[(k-j)T]$$

Please prove: the discrete-time convolution sum for functions f and g is equal to the product of discrete Fourier transforms of functions f and g , where the discrete Fourier Transformation of f is defined as

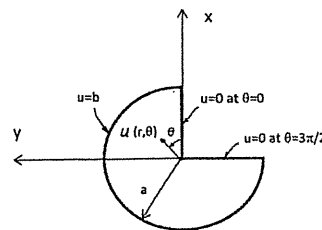
$$F(n/NT) = \sum_{k=0}^{N-1} f(kT)e^{-2\pi i kn/N}$$

3. Find the solution $u(r, \theta)$ in a three-quarter domain as given in the figure below. (15%)

$$\frac{\partial^2 u}{\partial r^2} + \frac{\partial u}{r \partial r} + \frac{\partial^2 u}{r^2 \partial \theta^2} = 0, \quad 0 < r < a, \quad 0 < \theta < 3\pi/2$$

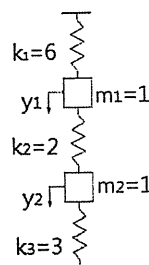
$$u(a, \theta) = b, \quad 0 < \theta < 3\pi/2$$

$$u(r, 0) = 0, \quad u(r, 3\pi/2) = 0, \quad 0 < r < a$$



4. Evaluate the Cauchy principal value of $\int_0^{\infty} \frac{x \sin x}{x^2+4} dx$ (15%)

5. Find the displacements $y_1(t)$ and $y_2(t)$ at any given time t for mass m_1 and mass m_2 , where $y_1(0) = 2, y_2(0) = -2, y_1'(0) = y_2'(0) = 0$ (15%)



6. Evaluate the integral of $F = [x^2, 2y^2, 3z^2]$ over the surface S that is the portion of a plane of $x + y + z = 2$ in the first octant. (15%)