

考試科目	微積分	所別	財政學系	考試時間	3月7日(日) 第四節
------	-----	----	------	------	-------------

每題 10 分，共 100 分

- Suppose that $\lim_{x \rightarrow 0} \frac{\sqrt{x^2 + x + 1} - (1 + ax)}{x^2} = b$, find $a + b$.
- Find $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{1}{2n + 3k}$.
- Let $f(x)$ be a positive differentiable function, and for any real numbers x, r , $f(rx) = (f(x))^r$. If $f(1) = 2$, find $f'(1)$.
- Let $g(x) = \int_x^{x^3} te^{t^2} dt$. Find $g''(0)$.
- Find the smallest positive number A such that $\ln x \leq Ax^3$ for all $x > 0$.
- Evaluate the following:
 - $\int_1^e x (\ln x)^2 dx$
 - $\int_{-1}^{\frac{3}{2}} x \llbracket x^2 \rrbracket dx$ where $\llbracket y \rrbracket$ denotes the Gaussian function, that is the largest integer that is less than or equal to y .
- Find the Maclaurin series for $f(x) = \frac{x}{(1-x)^2}$. Use this series to find the sum $\sum_{n=1}^{\infty} (n-1) \left(\frac{1}{2}\right)^n$.
- Solve the differential equation $y'' = 2y'$ with $y'(0) = y(0) = e$.
- Suppose that $z = f\left(\frac{x-y}{x+y}\right)$ for some differentiable function f . Show that $x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = 0$.
- Suppose that the rate $\frac{dp}{dt}$ at which the consumer price index (CPI) changes is proportional to p with constant inflation rate k , that is $\frac{dP}{dt} = kP$. Find $p(t)$ when $p(0) = p_0$. About how many years will it take the CPI to increase 50% if the inflation rate is 0.05?