

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Please solve the following differential equations: (5 points for each one)

A.  $y''' + 2y'' + 4y' + 8y = 6xe^{-2x}$

B.  $x^2y'' - 2xy' + 2y = x^4e^x$

C.  $y'' - 4ty' + 4y = 0; y(0) = 0, y'(0) = 10$

D.  $y \frac{dy}{dx} = x^2 + y^2 - x$

2. Please solve  $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$  for the following conditions. (15 points for each one)

A.  $\begin{cases} u(0, y) = 1, \lim_{x \rightarrow \infty} u(x, y) = 0, & 0 < y < 1 \\ \left. \frac{\partial u}{\partial y} \right|_{y=0} = 0, \left. \frac{\partial u}{\partial y} \right|_{y=1} = -u(x, 1), & x > 0 \end{cases}$

B.  $\begin{cases} u(0, y) = 0, u(\pi, y) = e^{-y}, & y > 0 \\ \left. \frac{\partial u}{\partial y} \right|_{y=0} = 0, & 0 < x < \pi \end{cases}$

3. Please solve  $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \sin x$  with  $\begin{cases} u(x, 0) = 400 + \sin x, & 0 < x < \pi \\ t > 0, u(0, t) = 400, u(\pi, t) = 200 \end{cases}$  (15 points)

4. If explicit finite difference method  $\frac{T_i^{n+1} - T_i^n}{\Delta t} = \frac{T_{i+1}^n - 2T_i^n + T_{i-1}^n}{\Delta x^2}$  is used for the partial

differential equation  $\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2}$ , (A) please derive the truncation error; (B) please derive the conditions for stability. (15 points)

5. The NOx emitted is 90% of NO and 10% of NO<sub>2</sub>. The emitted NO is converted into NO<sub>2</sub> by reaction with O<sub>3</sub> as  $NO + O_3 \rightarrow NO_2 + O_2$ , where the rate constant is  $2 \times 10^{-14}$  cm<sup>3</sup>/molecule sec. If the concentration of NO<sub>2</sub> is three times greater than that of NO when the O<sub>3</sub> concentration is 100 ppbv. Please estimate the distance of the emission source from the measurement site when wind speed is 2 m/sec. (20 points)