

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

注意：可以用中文或英文作答。

(1-7 題為單選題，每一題 4 分。請直接寫下選項，答錯不倒扣。)

1. What is the order of magnitude (數量級) of the Rydberg constant (雷德堡常數) in  $m^{-1}$ ?  
(a)  $10^5$  (b)  $10^6$  (c)  $10^7$  (d)  $10^8$  (e)  $10^9$
2. The ground state energy for an electron moving within an infinite square well of width  $a$  is found to be  $E_0$ . What is the new ground state energy if the width of the infinite square well becomes  $2a$ ?  
(a)  $0.25E_0$  (b)  $0.5E_0$  (c)  $E_0$  (d)  $2E_0$  (e)  $4E_0$
3. How many split lines will be observed for the single yellow line of Na atom in the normal Zeeman effect? Neglecting the effect of the spin-orbital coupling. (a) 1 (b) 2 (c) 3 (d) 4 (e) 5
4. Consider an ideal two-level paramagnet with identical magnetic dipoles under an external magnetic field  $B = 10$  Tesla. Each dipole contains a magnetic moment  $\mu = 10^{-23}$  J/T. What is the emitted light due to the transition from the excited state to the ground state?  
(a) X-ray (b) Ultraviolet (c) Visible light (d) Infrared (e) Microwave
5. In the following, which physical quantity or constant is not relevant to the Planck constant  $h$ ?  
(a) Wien's displacement constant (b) Stefan-Boltzmann constant (c) Rydberg constant (d) Rutherford scattering differential cross section (e) Compton wavelength
6. In 1911, Ehrenfest proposed the term of "ultraviolet catastrophe" to describe the disagreement between classical theory and an experimental result. Which was the mentioned experimental result?  
(a) Lyman series of hydrogen spectrum (b) Franck-Hertz experiment (c) Rutherford scattering experiment  
(d) Black-body radiation (e) Photoelectric effect experiment
7. Which experiment provided strong evidence for supporting Bohr's atomic model?  
(a) Stern-Gerlach experiment (b) Franck-Hertz experiment (c) Davison-Germer experiment  
(d) Compton scattering experiment (e) Photoelectric effect experiment

(8-17 題為問答題，每一題 5 分。請直接寫下答案，勿將計算過程寫在答案卷上。)

8. A 2.48 eV photon has a wavelength of 500 nm. At what energy will an X-ray photon have a wavelength of 1 Å?
9. Within the non-relativistic limit, a moving particle with kinetic energy of 1 keV has a wavelength of 0.5 Å, what is the kinetic energy of this moving electron which has a wavelength of 2 Å?
10. In the Compton scattering experiment, a photon with energy of 1 MeV is scattered by an electron at rest into an angle of  $60^\circ$ . The corresponding wavelength of the scattered photon is found to be 0.136 Å. What is the wavelength of the scattered photon into the same angle of  $60^\circ$  as the incident energy becomes 2 MeV?
11. Based on Bohr's model for a hydrogen atom, the circular motion of an electron with a constant speed  $v$  around a fixed radius  $a_B$  (Bohr radius) will produce a magnetic dipole moment of a Bohr magneton  $\mu_B$ . What is the ratio of the rotating speed  $v$  to the speed of light  $c$ ?

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12. How many split lines will be observed in the Stern-Gerlach experiment if an atom has a total orbital quantum number  $J = 3/2$ ?
13. In the Balmer series of hydrogen spectrum, the longest wavelength was found to be 656 nm. What is the second longest wavelength of the Balmer series?
14. A stick of length 2 m moves along its length with a speed of  $1.8 \times 10^8$  m/s relative to you. What is the length of the stick observed by you?
15. What is the value of the rest mass energy of an electron in the unit of MeV?
16. There are five non-interacting identical fermions in a cubic box with a volume of  $a^3$ . How many degeneracy states exist in the lowest energy state?
17. A system with  $N$  distinguishable particles which are distributed over two energy levels  $-\varepsilon_0$  and  $\varepsilon_0$ . Compute the population for the particles in the energy level of  $-\varepsilon_0$  at a fixed temperature  $T = \varepsilon_0/k$ .

(18-19 題為計算題。需詳列計算過程，僅列式正確或僅答案正確，則得部份分數。)

18. Use the Bohr-Sommerfeld quantization rule  $\oint p_x dx = nh$  to calculate the allowed energy level of a particle of mass  $m$  moving in a one-dimensional potential well  $V(x) = kx^2/2$ . Express your answer in terms of the mass  $m$ , the constant  $k$ , the Planck constant  $h$ , and the quantum number  $n$ . (10 分)
19. The ground state wave function  $\Psi_0(x, t)$  of a Hamiltonian  $\hat{H} = -\frac{\hbar^2}{2m} \frac{d^2}{dx^2} + \frac{m\omega_0^2}{2} x^2$  has a form  $\Psi_0(x, t) = A \exp(-Bx^2)$ . (a) Determine the constant  $A$  and  $B$ . (b) Evaluate the expectation value of  $x^2$ ,  $\langle x^2 \rangle$ . (c) What is the eigen-energy for the first excited state? Express your answers in terms of the mass of the electron  $m$ , the angular frequency  $\omega_0$ , and the reduced Planck constant  $\hbar$ . 第(c)小題請直接寫下答案，無需列計算過程。(12 分)

- Bohr radius  $a_B = 0.53 \times 10^{-10}$  m = 0.53 Å; Bohr magneton  $\mu_B = 0.93 \times 10^{-23}$  amp-m<sup>2</sup> (or joule/tesla)
- $g$ -factor of the electron spin  $g = 2$ ; Light speed  $c = 3 \times 10^8$  m/s;
- Magnetic permeability constant  $\mu_0 = 4\pi \times 10^{-7}$  webb/amp-m (or henry/m)
- Electron mass  $m = 9.11 \times 10^{-31}$  kg; Planck constant  $h = 6.63 \times 10^{-34}$  J sec
- Boltzmann constant  $k = 1.38 \times 10^{-23}$  J/K
- Wien's displacement law:  $\lambda_{max} T = 2.898 \times 10^{-3}$  m-K
- $\cos^2 \theta = (1 + \cos 2\theta)/2$ ;  $e^{+1} \sim 2.718$ ;  $e^{-1} \sim 0.368$ ;  $e^{-2} \sim 0.135$ ;
- $\int_{-\infty}^{\infty} e^{-ax^2} dx = \sqrt{\frac{\pi}{a}}$ .